Global NEST Journal, Vol 27, No 10, 07823

Evolutionary game analysis of collaborative mechanism in the third-party environmental pollution governance

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Received: 07/07/2025, Accepted: 19/10/2025, Available online: 11/11/2025
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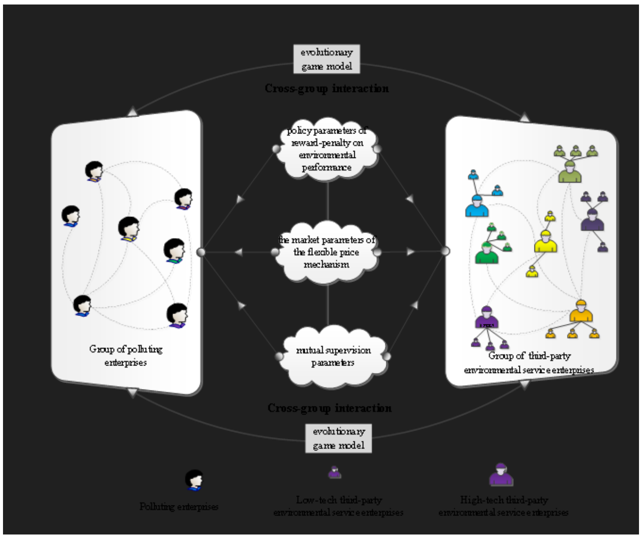
https://doi.org/10.30955/gnj.07823

Graphical abstract



Abstract

Achieving sustainable development through third-party environmental pollution governance (TEPG) necessitates effective collaboration among key stakeholders. This study constructs an evolutionary game model to analyze the strategic interactions between polluting enterprises (PEs) and third-party environmental service enterprises (TESEs). The model incorporates technological heterogeneity among TESEs and three critical mechanisms: government reward-penalty policies, market-based flexible pricing, and mutual supervision. Theoretical and simulation analysis reveal that the TEPG system exhibits multiple evolutionary pathways. A synergistic policy-market-supervision approach is essential to balance costs and benefits, guiding the system toward an ideal state of collaboration. The market price mechanism's heterogeneous effects must be positioned according to different TEPG development stages, supplemented by policy and supervision instruments that meet technological optimization conditions, to facilitate steering the TEPG market toward sustainable and high-quality development. Moreover, stimulating endogenous motivation for pollution control while strategically leveraging external resources significantly accelerates the adoption of collaborative behaviors among both PEs and TESEs.

Keywords: collaborative mechanism; evolution path; evolutionary game theory; third-party governance

1. Introduction

The dual carbon target represents the strategic objectives of carbon emission peaking and carbon neutrality. This target establishes four fundamental balanced relationships: between development and emission reduction, overall and local interests, long-term and short-term goals, and government and market roles (Dai *et al.* 2022). Specifically, the relationship between development and emission reduction can be transformed through main enterprises maintaining their development focus while third-party environmental service enterprises (TESEs) professionally handle emission reduction, enabling parallel pursuit of high-quality development and emission reduction. The whole-parts relationship can be optimized by embedding local emission reduction resources at polluting nodes, with the main business as the core, through third-party governance models that ensure efficient and targeted emission reduction management. The long-term-short-term relationship is addressed through enterprises' sustained development of their main business while transferring short-term pollution control pressures to TESEs, thus achieving simultaneous long-term performance and short-term emission reduction goals.

The relationship between the whole and the parts can be transformed into embedding local emission reduction resources for polluting nodes with the main business of the enterprise as the overall core, that is, using third-party governance models to form efficient and targeted emission reduction management. The relationship between long-term and short-term is transformed into the sustained independent development of the economic main business by enterprises, and the short-term pressure of pollution control or emission reduction is shifted to TESEs to jointly undertake, achieving the goal of coexistence of long-term performance and short-term emission reduction construction. The government-market relationship under third-party governance facilitates effective multi-stakeholder interaction and high-quality implementation. By aligning with government carbon strategies and market demands, an effective third-party governance model can be established. Essentially, governments aim to balance economic and environmental development for enterprises, but limitations in main business operations and the high costs of traditional pollution control equipment often hinder this balance. Consequently, market demands have driven the formation of professional TESEs. Therefore, exploring third-party governance models contributes significantly to promoting and achieving the dual carbon targets.

In response to escalating environmental pollution and ecological degradation, China has implemented a series of laws and regulations to advance its national ecological civilization (Gao *et al.* 2022; Tian *et al.* 2019; Li and Li 2019; Kong *et al.* 2024). By 2020, China had established 462 local environmental protection laws, 152 regulations, and 22 standards governing environmental quality and pollutant discharge. To enforce these policies effectively and prevent collusion between local governments and enterprises, China has progressively established a comprehensive environmental supervision system. This system incorporates environmental interviews, regional inspections, and central inspections, designed to overcome shortcomings of previous regulations through coordinated party-government joint actions (Zhang *et al.* 2018; Chen *et al.* 2020; Zhang *et al.* 2024). During the transition from regulation policies to a supervision system, the consistent emphasis on strict command-and-control measures has inadvertently led to the alienation and misinterpretation of pollution prevention systems. This approach has hindered the development of durable environmental protection mechanisms and the reversal of ongoing ecological degradation (Chong and Sun 2020). The high-pressure, control-centered management model exhibits a “tight-loose” enforcement pattern that proves inadequate for thorough and sustainable adaptation to current environmental challenges. Academics and policymakers have increasingly explored multi-agent interactive models (Liu 2015; Cao *et al.* 2023; Xing 2023) featuring joint participation and mutual consultation to enhance implementation quality and engagement for sustainable environmental governance. As an exemplary interactive model, third-party environmental pollution governance (TEPG) has solidified its role in expanding China’s environmental protection industry. According to the 2020 China Ecological Environment Statistics Yearbook, the industry’s operating revenue reached 1.95 trillion yuan, with environmental services accounting for 33.3% of the total. As an innovative approach within the environmental services sector, the TEPG model has been widely adopted, proving effective in pollution control and facilitating industrial transformation and upgrading.

In December 2014, the General Office of the State Council of China issued the “Opinions on Promoting Third-Party Environmental Pollution Governance,” which outlined the developmental direction and goals for the TEPG model, cementing its crucial role in China’s environmental governance. Subsequently, key governmental departments, including the National Development and Reform Commission and the Ministry of Ecology and Environment, have issued multiple documents emphasizing the necessity of vigorously promoting this innovative TEPG model and the urgency of its effective implementation. These documents include the 2019 “Announcement on Corporate Income Tax Policy Issues for Third-Party Enterprises Engaged in Pollution Prevention,” the 2021 “Implementation Plan for Special Actions on Green Development in National High-Tech Zones,” and the 2022 “Guiding Opinions on Accelerating the Promotion of Urban Environmental Infrastructure Construction.” The proposal of TEPG marks a pivotal shift from the traditional, government-led, one-way administrative model of pollution control to a contract-based mode involving multiple agents. This transformation underscores a more dynamic and collaborative approach to environmental management. In this model, the government transitions from a direct controller to a facilitator, guiding and encouraging multiple agents to participate actively in environmental governance. Furthermore, this model signifies a transition from “the polluter manages” to “the polluter pays.” Specifically, polluting enterprises (PEs) hire independent TESEs for pollutant disposal based on their technical expertise and through contractual payment agreements (Du *et al.* 2015; Yang *et al.* 2024). The TEPG model inherently involves the interaction of interests and demands between PEs and TESEs. On one hand, the negative externalities of environmental pollution diminish the motivation for PEs with limited treatment capabilities to adopt environmental protection behaviors. The high-cost investment in environmental governance often contradicts the profit-maximization principle pursued by PEs as rational economic agents, making the implementation of proactive environmental behaviors challenging (Xu *et al.* 2019). On the other hand, TESEs possess strong professional expertise in pollution treatment technology and operational management. They can achieve economies of scale, thereby offsetting the negative externalities of environmental pollution and fulfilling the fundamental goal of environmental protection (Zhou *et al.* 2019). Therefore, a practical need for cooperation between PEs and TESEs is evident. However, from the perspective of classical economics, which adheres to the cost-benefit principle, dual moral hazards exist in the pollution control process. These hazards can lead to deviations from the intended collaborative governance path, thereby hindering the achievement of desired environmental governance outcomes.

Collaborative governance in the environmental protection field is currently a focal point of academic interest. Although collaborative governance targets (e.g., air and water pollution) and the diverse actors involved exhibit heterogeneity, the consistency of governance goals provides valuable referential insights across studies. Particularly, transboundary water pollution and regional air pollution exhibit ecological characteristics—including negative externalities, public goods nature, and fluidity—that prevent individual governance entities from achieving environmental goals independently. Consequently, the challenge of promoting collaborative governance to circumvent the free-rider problem has garnered widespread attention (Zhang *et al.* 2018; Yang *et al.* 2021). Research on transboundary river basin pollution, for instance, examines collaborative governance stability (Li and Guo 2019; Zhao *et al.* 2024) and evaluates organizational interventions (Lu *et al.* 2022; Chien *et al.* 2018), performance-based incentives, and ecological compensation mechanisms (Wang *et al.* 2022; Yang *et al.* 2021). Similarly, air pollution studies have investigated administrative orders and economic instruments (Yang *et al.* 2021), confirming the positive effects of government policies (Kim *et al.* 2022; Jiao *et al.* 2021), higher-level constraints (Meng *et al.* 2021), ecological compensation (Lu *et al.* 2019; Jia *et al.* 2021), and emission trading markets (Klenert *et al.* 2018). While these studies primarily explore factors influencing participant behavior and multi-agent interactions, their applicability within the TEPG model requires further examination. Existing TEPG research has mainly addressed legal dilemmas in environmental service contracts (Tang and Wei 2020; Ren *et al.* 2021) and participant behavioral evolution (Xu *et al.* 2019; Zheng *et al.* 2021). For example, Tang and Wei (Tang and Wei 2020), and Ren (Ren 2021) emphasize the legal urgency of clarifying contractual responsibilities in TEPG practices. However, regulating corporate environmental behavior in TEPG involves not only legal constraints but also complex market interactions among diverse stakeholders. Previous studies have employed evolutionary game models (Xu *et al.* 2019) and stochastic differential game models (Zheng *et al.* 2021) to analyze multi-agent negotiations and constraints within TEPG, exploring cooperation possibilities. However, the former study (Xu *et al.* 2019) has not considered information asymmetry regarding third-party enterprises' technical capabilities—a critical factor affecting governance effectiveness, contract fulfillment, and cooperation mechanisms. While the latter study (Zheng *et al.* 2021) accounted for variations in technical R&D capabilities, it primarily focused on evaluating the performance and feasibility of participant cooperation. Building on this foundation, our study distinguishes types of technical information available to TESEs and investigates boundary conditions for optimizing TEPG's technical environment. This research aims to explore collaborative pollution control trajectories and provide a managerial framework for developing high-tech TEPG markets. Given its foundation in limited rationality assumptions, evolutionary game theory has proven valuable for studying decision-making in mutual interest contexts (Liu *et al.* 2015; Guo *et al.* 2024) and has been widely applied in environmental governance research (Sun *et al.* 2021; Chong and Sun 2020; Guo *et al.* 2024; Lei *et al.* 2024). Accordingly, this paper employs evolutionary game theory to examine collaborative governance pathways, constructing a TEPG model involving PEs and TESEs. In summary, this study makes three primary contributions: (1) This paper introduces a typology of TESEs based on their technical characteristics, explicitly modeling technological heterogeneity as a core element of the evolutionary game. This approach goes beyond existing models that treat TESEs as a homogeneous group. (2) This paper develops a more comprehensive evolutionary game model that integrates three pivotal driving mechanisms: government reward-penalty policies, market-based flexible pricing, and mutual supervision mechanisms. This enables the analysis of their synergistic effects on collaborative governance pathways. (3) This paper identifies the conditions under which the system evolves toward an optimal state. Also, the study elucidates the differential roles of market mechanisms and provides a clear decision-making framework for policymakers to cultivate a sustainable TEPG market dominated by high-tech service providers.

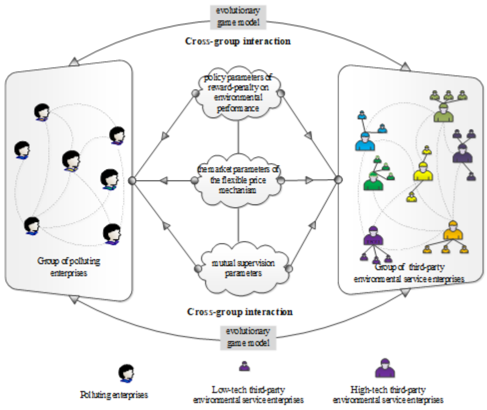


Figure 1. The sketch of TEPG

2. Basic Model Assumptions and Parameter Descriptions

Given that the primary incentive for both PEs and TESEs is economic benefit, any excessive compression of profit margins or disruption of interest balance could hinder the achievement of environmental management objectives (Xu *et al.* 2019). As key participants in TEPG, these two types of enterprises exhibit the characteristics of "economic man" while maintaining distinct interests. The TEPG model incorporates specific guiding instruments designed to balance and secure their respective benefits while achieving environmental protection targets. This design fosters a dynamic, self-adaptive collaborative governance process through mutual learning, communication, and negotiation between the two parties. Building on the collaborative governance literature and considering TEPG characteristics, this paper proposes three types of controllable parameters that directly drive strategic adjustments: government reward-penalty policy parameters for environmental performance, market-based flexible pricing mechanism parameters, and mutual supervision parameters to mitigate moral hazard (as shown in **Figure 1**).

(1) Government reward-penalty policy parameters for environmental performance serve as key mechanisms to directly influence governance behavior. These parameters

effectively coordinate negative environmental externalities by providing clear incentives and penalties, directly impacting environmental outcomes (Yang *et al.* 2021). This mechanism also signifies a shift in the government's role within TEPG from controller to facilitator, guiding and encouraging multi-agent engagement in environmental governance, as highlighted in the Introduction.

(2) Given that TEPG is fundamentally market-oriented, introducing flexible pricing mechanism parameters is essential to explore their effectiveness. In 2014, China's Ministry of Environmental Protection issued an announcement abolishing eight standards under the "Classification and Grading Standards for Operational Qualifications of Environmental Pollution Control Facilities (1st Edition)." This policy lowered the entry barriers for professional service enterprises, leading to an influx of environmental service enterprises with varying technical levels and treatment capabilities into the TEPG market. Some enterprises winning projects through low-price competition may gain short-term profits but often deliver poor environmental performance, fostering disorderly competition and wasting resources. To counteract this, flexible pricing mechanisms are essential to promote market entry by technologically advanced and environmentally committed enterprises (Chen *et al.* 2019).

Table 1. Related parameters and definitions

Parameters	Definitions
P^*	Predetermined pollutant stock.
P_0	The volume of pollutants treated by TESEs commissioned by PEs.
P_1	The volume of pollutants illicitly discharged by PEs.
θ^z	$z \in \{h,l\}$, where it indicates there exist θ^h proportion of high-tech TESEs if $z=h$ or θ^l proportion of low-tech TESEs if $z=l$.
P_j^z	$z \in \{h,l\}$ and $j \in \{P,N\}$, where it indicates the genuine treatment volume by high-tech TESEs with fulfilling positive pollution control if $z=h$ and $j=P$; other scenarios follow similarly.
Δ	It indicates the positive impact of active cooperation from PEs. Given that the treatment amount of pollutants by TESEs depends not only on their management level but also critically on the cooperation of the PEs, it is assumed that the treatment volume will increase to ΔP_j^z when PEs cooperate with environmental governance.
C_k	$k \in \{P,N\}$, where it denotes the costs incurred by TESEs for positive treatment if $k=P$ or negative treatment if $k=N$.
C_{PE}	To mitigate moral hazard in TESEs, PEs supervise them during the contract term. This parameter represents the supervisory costs.
f_{PE}	Probability of PEs being caught for excessive emissions.
η_{PE}	The punishment imposed on PEs when caught excessive emissions.
C_{TE}	To prevent excessive emissions by PEs, TESEs will supervise during the contract period, incurring supervision costs.
f_{TE}	Probability of TESEs being caught for negative pollution control.
η_{TE}	Punishment imposed on TESEs found applying negative pollution control.
β_1	Policy-based reward-penalty allocated to TESEs.
β_2	Policy-based reward-penalty allocated to PEs.
$R(P_0)$	Benefits for PEs corresponding to the emission of P_0 pollutants.
$R(P_0+P_1)$	Benefits for PEs corresponding to the emission of P_0+P_1 pollutants.
Parameter relationships	Definitions
$B_\psi^z = \begin{cases} B_j^z = B_0 + B_1 P_j^z, \psi = j \\ B_{\Delta j}^z = B_0 + B_1 \Delta P_j^z, \psi = \Delta j \end{cases}$	According to assumption 5 and Ref. (Chen *et al.* 2019), the payment fees of PEs can be either fixed or based on a flexible pricing mechanism. Accordingly, we establish a linear payment function B_ψ^z to represent this fee structure. Based on the definition of parameter P_j^z, B_ψ^z is divided into two categories, where $B_{\Delta j}^z$ denotes the costs paid by PEs under the influence of positive effects Δ, B_j^z reflects the costs without factoring in these positive effects. B_0 represents the fixed payments for the delegated pollution treatment volume and B_1 indicates payment with flexible pricing for the contracted pollution control volume, where variable payments are made according to the actual amount of pollution treated by TESEs.
Variables	Definitions
x	The proportion of TESEs that choose positive pollution control strategy.
y	The proportion of PEs actively complying with and meeting emissions commitments in environmental service contracts.

(3) Environmental performance reward-penalty systems underscore the necessity for all participants to fulfill responsibilities and meet environmental targets. Deviating from this collaborative path may lead to the failure of achieving established environmental objectives. As noted in the Introduction, both TESEs and PEs face dual moral hazards during pollution control, potentially diverting them from the intended governance path. Consequently, developing supervisory behaviors is crucial to maintain TEPG alignment, ensure contractual adherence, and mitigate risks from informational disadvantages in disputes. Therefore, this analysis incorporates mutual supervision parameters designed to constrain behaviors and reinforce compliance within TEPG.

Accordingly, this paper proposes the following assumptions to clarify the research problem.

Assumption 1. The game participants in TEPG collaborative governance comprise TESEs i and PEs j. For TESEs, their strategic choice involves whether to actively engage in pollution treatment. Thus, their strategy space is defined as {positive pollution control, negative pollution control}, denoted as $TE_i = \{PC, NC\}$. As both the emission source and key participants, PEs should ideally establish trust relationships with TESEs and honor payment agreements. However, driven by cost-reduction and profit-maximization motives, they may deviate from cooperation and choose excessive emissions (Zheng *et al.* 2021). Therefore, the strategic options for PEs are defined as {keeping emissions promised, breaching emissions promised}, denoted as $PE_j = \{KP, BP\}$.

Assumption 2. To meet local environmental protection targets, governments set predetermined pollutant stock (denoted as P^*) before assessment periods. Based on this target, governments implement environmental performance reward-penalty mechanisms. After verification of actual emissions, reward-penalty are applied according to the comparison between the actual emissions and the predetermined pollutant stock.

Assumption 3. Given that TESEs' technical capabilities are inherent and not fully observable to PEs, we assume two types exist in the market (Chen *et al.* 2019): high-tech TESEs and low-tech TESEs, with market shares of θ and $1-\theta$ respectively.

Assumption 4. Under collaborative governance, TESEs and PEs share responsibility for pollution control outcomes, creating mutual supervision constraints. The monitoring costs are C_{PE} for *PEs* and C_{TE} for TESEs. The detection probabilities are f_{PE} for detecting PEs breaching emissions and f_{TE} for detecting TESEs negative pollution control, with corresponding penalties η_{PE} and η_{TE}.

Assumption 5. Pollution treatment fees can be structured in two ways: (1) fixed payment (denoted as B_0) for contracted disposal amounts, (2) flexible pricing based on actual treatment quantity with pricing coefficient B_1.

All model parameters are systematically summarized in **Table 1.**

3. Analysis of collaborative pollution treatment on the TEPG game model

3.1. Model construction and analysis of equilibrium points

Based on the defined parameters, we construct a TEPG game model that incorporates the technical types of TESEs, with the corresponding payoff matrix shown in **Table 2.** Here, $U_1(\theta^h, PC, KP)$ represents the payoff when a high-tech TESE adopts positive pollution control strategy PC and the PE chooses the keeping emissions promised strategy KP. Other strategy combinations follow similarly.

Table 2. Payoffs for strategy combinations of PEs and TESEs.

Strategy Combinations	**Payoff**
$U_1(\theta^h, PC, KP)$	$B^h_{\Delta P} - C_P + \beta_1[P^* - (P_0 - \Delta P^h_P)] - C_{TE}$, $R(P_0) - B^h_{\Delta P} + \beta_2[P^* - (P_0 - \Delta P^h_P)] - C_{PE}$
$U_2(\theta^h, NC, KP)$	$B^h_{\Delta N} - C_N + \beta_1[P^* - (P_0 - \Delta P^h_N)] - C_{TE} - f_{TE}\eta_{TE}(P_0 - \Delta P^h_N)$, $R(P_0) - B^h_{\Delta N} + \beta_2[P^* - (P_0 - \Delta P^h_N)] - C_{PE} + f_{TE}\eta_{TE}(P_0 - \Delta P^h_N)$
$U_3(\theta^l, PC, KP)$	$B^l_{\Delta P} - C_P + \beta_1[P^* - (P_0 - \Delta P^l_P)] - C_{TE}$, $R(P_0) - B^l_{\Delta P} + \beta_2[P^* - (P_0 - \Delta P^l_P)] - C_{PE}$
$U_4(\theta^l, NC, KP)$	$B^l_{\Delta N} - C_N + \beta_1[P^* - (P_0 - \Delta P^l_N)] - C_{TE} - f_{TE}\eta_{TE}(P_0 - \Delta P^l_N)$, $R(P_0) - B^l_{\Delta N} + \beta_2[P^* - (P_0 - \Delta P^l_N)] - C_{PE} + f_{TE}\eta_{TE}(P_0 - \Delta P^l_N)$
$U_5(\theta^h, PC, BP)$	$B^h_P - C_P + \beta_1[P^* - (P_0 + P_1 - P^h_P)] - C_{TE} + f_{PE}\eta_{PE}P_1$, $R(P_0 + P_1) - B^h_P + \beta_2[P^* - (P_0 + P_1 - P^h_P)] - C_{PE} - f_{PE}\eta_{PE}P_1$
$U_6(\theta^h, NC, BP)$	$B^h_N - C_N + \beta_1[P^* - (P_0 + P_1 - P^h_N)] - C_{TE} - f_{TE}\eta_{TE}(P_0 - P^h_N) + f_{PE}\eta_{PE}P_1$,

	$R\left(P_0+P_1\right)-B_N^h+\beta_2[P^*-\left(P_0+P_1-P_N^h\right)]-C_{PE}+f_{TE}\eta_{TE}\left(P_0-P_N^h\right)-f_{PE}\eta_{PE}P_1$
$U_7(\theta^l,PC,BP)$	$B_P^l-C_P+\beta_1[P^*-\left(P_0+P_1-P_P^l\right)]-C_{TE}+f_{PE}\eta_{PE}P_1,$ $R\left(P_0+P_1\right)-B_P^l+\beta_2[P^*-\left(P_0+P_1-P_P^l\right)]-C_{PE}-f_{PE}\eta_{PE}P_1$
$U_8(\theta^l,NC,BP)$	$B_N^l-C_N+\beta_1[P^*-\left(P_0+P_1-P_N^l\right)]-C_{TE}-f_{TE}\eta_{TE}\left(P_0-P_N^l\right)+f_{PE}\eta_{PE}P_1,$ $R\left(P_0+P_1\right)-B_N^l+\beta_2[P^*-\left(P_0+P_1-P_N^l\right)]-C_{PE}+f_{TE}\eta_{TE}\left(P_0-P_N^l\right)-f_{PE}\eta_{PE}P_1,$

According to **Table 2**, the payoff for TESEs choosing PC (E_{PC}) and NC (E_{NC}), and the average payoff $\overline{E_{TE}}$, are as follows:

$$\begin{cases} E_{PC}=\theta\{-C_P-C_{TE}+y[B_{\Delta P}^h+\beta_1[P^*-(P_0-\Delta P_P^h)]]+(1-y)[B_P^h+\beta_1[P^*-(P_0+P_1-P_P^h)] \\ \quad +f_{PE}\eta_{PE}P_1]\}+(1-\theta)\{-C_P-C_{TE}+y[B_{\Delta P}^l+\beta_1[P^*-(P_0-\Delta P_P^l)]+(1-y)[B_P^l \\ \quad +\beta_1[P^*-(P_0+P_1-P_P^l)]+f_{PE}\eta_{PE}P_1]\} \\ E_{NC}=\theta\{-C_N-C_{TE}+y[B_{\Delta N}^h+\beta_1[P^*-(P_0-\Delta P_N^h)]-f_{TE}\eta_{TE}(P_0-\Delta P_N^h)]+(1-y)[B_N^h+\beta_1[P^*-(P_0 \\ \quad +P_1-P_N^h)]-f_{TE}\eta_{TE}\left(P_0-P_N^h\right)+f_{PE}\eta_{PE}P_1]\}+(1-\theta)\{-C_N-C_{TE}+y[B_{\Delta N}^l+\beta_1[P^*-(P_0-\Delta P_N^l) \\ \quad -f_{TE}\eta_{TE}\left(P_0-\Delta P_N^l\right)]+(1-y)[B_N^l+\beta_1[P^*-(P_0+P_1-P_N^l)]-f_{TE}\eta_{TE}\left(P_0-P_N^l\right)+f_{PE}\eta_{PE}P_1\} \\ \overline{E_{TE}}=xE_{PC}+\left(1-x\right)E_{NC} \end{cases} \tag{1}$$

The payoff for PEs choosing KP (E_{KP}) and BP (E_{BP}), and the average payoff $\overline{E_{PE}}$, are as follows:

$$\begin{cases} E_{KP}=\theta\{R(P_0)-C_{PE}+x[-B_{\Delta P}^h+\beta_2[P^*-(P_0-\Delta P_P^h)]]+(1-x)[-B_{\Delta N}^h+\beta_2[P^*-(P_0-\Delta P_N^h)] \\ \quad +f_{TE}\eta_{TE}\left(P_0-\Delta P_N^h\right)]\}+(1-\theta)\{R(P_0)-C_{PE}+x[-B_{\Delta P}^l+\beta_2[P^*-(P_0-\Delta P_P^l)]]+(1-x)[-B_{\Delta N}^l \\ \quad +\beta_2[P^*-(P_0-\Delta P_N^l)]+f_{TE}\eta_{TE}\left(P_0-\Delta P_N^l\right)]\} \\ E_{BP}=\theta\{R(P_0+P_1)-C_{PE}-f_{PE}\eta_{PE}P_1+x[-B_P^h+\beta_2[P^*-(P_0+P_1-P_P^h)]]+(1-x)[-B_N^h \\ \quad +\beta_2[P^*-(P_0+P_1-P_N^h)]+f_{TE}\eta_{TE}\left(P_0-P_N^h\right)]\}+(1-\theta)\{R(P_0+P_1)-C_{PE}-f_{PE}\eta_{PE}P_1 \\ \quad +x[-B_P^l+\beta_2[P^*-(P_0+P_1-P_P^l)]]+(1-x)[-B_N^l+\beta_2[P^*-(P_0+P_1-P_N^l)]+f_{TE}\eta_{TE}\left(P_0-P_N^l\right)]\} \\ \overline{E_{PE}}=yE_{KP}+\left(1-y\right)E_{BP} \end{cases} \tag{2}$$

Hence, integrating Eq. (1) and Eq. (2) and setting $P_d^z=P_P^z-P_N^z\left(z\in\{h,l\}\right)$, the two-dimensional dynamic system (abbreviated as system (＊)) for collaborative pollution treatment by PEs and TESEs can be described by the following equation:

$$\begin{cases} F(x)=\dfrac{dx}{dt}=x\left(E_{PC}-\overline{E_{TE}}\right)=x\left(1-x\right)\{-C_P+C_N+(B_1+\beta_1)[\theta P_d^h+(1-\theta)P_d^l]+f_{TE}\eta_{TE}[P_0- \\ \quad \theta P_N^h-(1-\theta)P_N^l]+y\left[(B_1+\beta_1)(\Delta-1)[\theta P_d^h+(1-\theta)P_d^l]-f_{TE}\eta_{TE}(\Delta-1)[\theta P_N^h+(1-\theta)P_N^l]\right]\} \\ G(y)=\dfrac{dy}{dt}=y\left(E_{KP}-\overline{E_{PE}}\right)=y\left(1-y\right)\{R(P_0)-R(P_0+P_1)+(f_{PE}\eta_{PE}+\beta_2)P_1+B_1(1-\Delta)[\theta P_N^h \\ \quad +(1-\theta)P_N^l]+(\beta_2-f_{TE}\eta_{TE})\ (\Delta-1)[\theta P_N^h+(1-\theta)P_N^l]+x[(\Delta-1)(\beta_2-1)[\theta P_d^h+(1-\theta)P_d^l] \\ \quad +f_{TE}\eta_{TE}(\Delta-1)[\theta P_N^h+(1-\theta)P_N^l]]\} \end{cases} \tag{3}$$

According to evolutionary game theory, PEs and TESEs will eventually reach a stable state in the process of continuously adjusting strategies, thereby ceasing the evolution of decision-making behaviors, i.e., $F(x)$=0 and $G(y)$=0. Consequently, Proposition 1 is as follows:

Proposition 1. System (＊) has four pure strategy equilibrium points, namely (0,0), (0,1), (1,0), (1,1), and one mixed equilibrium point, namely (x*, Y*),

$$0<x^*=-\{R(P_0)-R(P_0+P_1)+(f_{PE}\eta_{PE}+\beta_2)P_1+(\beta_2-f_{TE}\eta_{TE}-B_1)$$
$$(\Delta-1)[\theta P_N^h+(1-\theta)P_N^l)]\}/\{(\Delta-1)(\beta_2-1)$$
$$[\theta P_d^h+(1-\theta)P_d^l]+f_{TE}\eta_{TE}(\Delta-1)[\theta P_N^h+(1-\theta)P_N^l)]\}<1;$$
$$0<y^*=-\{-C_P+C_N+(B_1+\beta_1)[\theta P_d^h+(1-\theta)P_d^l]$$
$$+f_{TE}\eta_{TE}[P_0-\theta P_N^h-(1-\theta)P_N^l]\}/\{(B_1+\beta_1)(\Delta-1)$$
$$[\theta P_d^h+(1-\theta)P_d^l]-f_{TE}\eta_{TE}(\Delta-1)[\theta P_N^h+(1-\theta)P_N^l]\}<1$$

As indicated by Proposition 1, the evolutionary dynamic system contains multiple equilibrium solutions. The optimal equilibrium occurs when TESEs actively engage in pollution control while PEs honor their emission commitments (i.e., the (1,1) equilibrium point). Suboptimal equilibrium solutions exist where only one party actively participates: either PEs alone or TESEs alone (i.e., the (0,1) and (1,0) equilibrium points). The worst equilibrium solution occurs when neither party cooperates on environmental commitments. Whether these equilibrium points become Evolutionary Stable Strategies (ESS) requires further analysis using Lyapunov's first method proposed by Friedman *et al.* (1991).

3.2. Evolutionary path analysis of collaborative pollution treatment

According to the equilibrium point stability analysis method proposed by Friedman, whether an equilibrium point in a two-dimensional dynamical system becomes an ESS can be determined through the local stability analysis of the system's Jacobian matrix. The Jacobian matrix for the system (*) proposed in this paper is as follows:

$$J = \begin{bmatrix} \dfrac{\partial F(x)}{\partial x} & \dfrac{\partial F(x)}{\partial y} \\ \dfrac{\partial G(y)}{\partial x} & \dfrac{\partial G(y)}{\partial y} \end{bmatrix} = \begin{bmatrix} F_1 & F_2 \\ G_1 & G_2 \end{bmatrix} \tag{4}$$

Substituting the equilibrium points from Proposition 1 into Eq. (4), if it satisfies det J > 0 and trJ < 0, then the equilibrium point can be identified as an ESS (Zhou *et al.* 2019). Therefore, by taking the partial derivatives of $F(x)$ and $G(y)$ with respect to x and y in system (*), and setting $\phi_N = P_N^l + \theta(P_N^h - P_N^l)$ and $\phi_d = P_d^l + \theta(P_d^h - P_d^l)$, we obtain

$$F_1 = (1-2x)\{-C_P + C_N + (B_1 + \beta_1)\phi_d + f_{TE}\eta_{TE}(P_0 - \phi_N) + y[(B_1 + \beta_1)(\Delta - 1)\phi_d - f_{TE}\eta_{TE}(\Delta - 1)\phi_N]\},\ F_2 = x(1-x)[(B_1 + \beta_1)(\Delta - 1)\phi_d - f_{TE}\eta_{TE}(\Delta - 1)\phi_N],\ G_1 = y(1-y)[(\Delta - 1)(\beta_2 - 1)\phi_d + f_{TE}\eta_{TE}(\Delta - 1)\phi_N],\ G_2 = (1-2y)\{R(P_0) - R(P_0 + P_1) + (f_{PE}\eta_{PE} + \beta_2)P_1 + (\beta_2 - f_{TE}\eta_{TE} - B_1)(\Delta - 1)\phi_N + x[(\Delta - 1)(\beta_2 - 1)\phi_d + f_{TE}\eta_{TE}(\Delta - 1)\phi_N]$$

Based on this, the following proposition can be derived by calculating det J and trJ.

Table 3. Stability analysis of equilibrium points in scenarios 1-1, 1-2, and 1-3.

equilibrium points	$C_P - C_N < \min\{(B_1 + \beta_1)\phi_d + f_{TE}\eta_{TE}(P_0 - \phi_N), \Delta(B_1 + \beta_1)\phi_d + f_{TE}\eta_{TE}(P_0 - \Delta\phi_N)\}$								
	$R(P_0 + P_1) - R(P_0) - (f_{PE}\eta_{PE} + \beta_2)P_1 < (\beta_2 - f_{TE}\eta_{TE} - B_1)(\Delta - 1)\phi_N$			$(\beta_2 - f_{TE}\eta_{TE} - B_1)(\Delta - 1)\phi_N < R(P_0 + P_1) - R(P_0) - (f_{PE}\eta_{PE} + \beta_2)P_1 < (\beta_2 - B_1)(\Delta - 1)\phi_N + (\beta_2 - 1)(\Delta - 1)\phi_d$			$R(P_0 + P_1) - R(P_0) - (f_{PE}\eta_{PE} + \beta_2)P_1 > (\beta_2 - B_1)(\Delta - 1)\phi_N + (\beta_2 - 1)(\Delta - 1)\phi_d$		
	Scenario 1-1			**Scenario 1-2**			**Scenario 1-3**		
	det *J*	**tr*J***	**stability**	**det *J***	**tr*J***	**stability**	**det *J***	**tr*J***	**stability**
(0,0)	+	+	unstable	–	?	saddle point	–	?	saddle point
(0,1)	–	?	saddle point	+	+	unstable	+	+	unstable
(1,0)	–	?	saddle point	–	?	saddle point	+	–	*ESS*
(1,1)	+	–	*ESS*	+	–	*ESS*	–	?	saddle point
(x*, y*)	the point is meaningless under this condition			the point is meaningless under this condition			the point is meaningless under this condition		

Proposition 2. (1) In scenarios 1-1, 1-2, and 2-1, only ESS (1,1) exists in the system (*). (2) In scenarios 1-3 and 2-6, only ESS (1,0) exists. In scenarios 2-2 and 3-1, only ESS (0,1) exists. (3) In scenarios 2-5, 3-2, and 3-3, only ESS (0,0) exists. (4) In scenario 2-3, ESS (0,0) and ESS (1,1) coexist in the system (*).

Proof. Given the range of values for x and y, it is evident that any initial and evolutionary points are meaningful only within the two-dimensional space W = {(x, y) |0 ≤ x ≤ 1, 0 ≤ y ≤ 1}, otherwise, this equilibrium point is meaningless (Hosseini-Motlagh *et al.* 2022). Consequently, this allows for the determination of the stability of each equilibrium point in the system (*), as shown in **Tables 3 to 5**, with the corresponding evolutionary phase diagram of the system illustrated in **Figure 2**.

For TESEs, when PEs choose excessive emissions, the strategic choice of TESEs depends on the cost difference C_P–C_N between active and passive management, the expected profit difference $(B_1 + \beta_1)\phi_d$ from market and policy mechanisms, and the supervision punishment $f_{TE}\eta_{TE}(P_0 - \phi_N)$ from PEs. Specifically, TESEs tend to choose the NC strategy if $(B_1 + \beta_1)\phi_d + f_{TE}\eta_{TE}(P_0 - \phi_N) < C_P - C_N$; otherwise, they lean towards PC strategy. Similarly, when PEs choose to comply with regulations and actively fulfill their obligations, even with adjusted expected profit $\Delta(B_1 + \beta_1)\phi_d$ and supervision penalty $f_{TE}\eta_{TE}(P_0 - \Delta\phi_N)$, TESEs will still opt for the NC strategy if $\Delta(B_1 + \beta_1)\phi_d + f_{TE}\eta_{TE}(P_0 - \Delta\phi_N) > C_P - C_N$ is not satisfied. For PEs, when TESEs choose passive management, their strategic choice depends on the benefits of exceeding pollutant emissions $R(P_0 + P_1) - R(P_0)$, the supervision punishment $(f_{PE}\eta_{PE} + \beta_2)P_1$ from TESEs, the supervision benefits $f_{TE}\eta_{TE}(\Delta - 1)\phi_N$ from punishing TESEs' passive management, and the expected profits $(\beta_2 - B_1)(\Delta - 1)\phi_N$ for TESEs' negative governance based on market and policy instruments.

If $(\beta_2 - f_{TE}\eta_{TE} - B_1)(\Delta-1)\phi_N > R(P_0+P_1) - R(P_0) - (f_{PE}\eta_{PE}+\beta_2)P_1$, PEs tend to choose the KP strategy, otherwise, they prefer to BP strategy. When the TEPG market shows promising prospects and TESEs choose active pollutants disposal, the strategy of PEs hinges on the benefits from excessive emissions $R(P_0+P_1)-R(P_0)$, penalties for excessive emissions $(f_{PE}\eta_{PE}+\beta_2)P_1$, environmental performance benefits from actively managed TESEs $(\beta_2-1)(\Delta-1)\phi_P$, and investment costs $(B_1-1)(\Delta-1)\phi_N$ for TESEs that manage passively. PEs will tend to the KP strategy only if $(\beta_2-1)(\Delta-1)\phi_P-(B_1-1)(\Delta-1)\phi_N > R(P_0+P_1)-R(P_0)-(f_{PE}\eta_{PE}+\beta_2)P_1$.

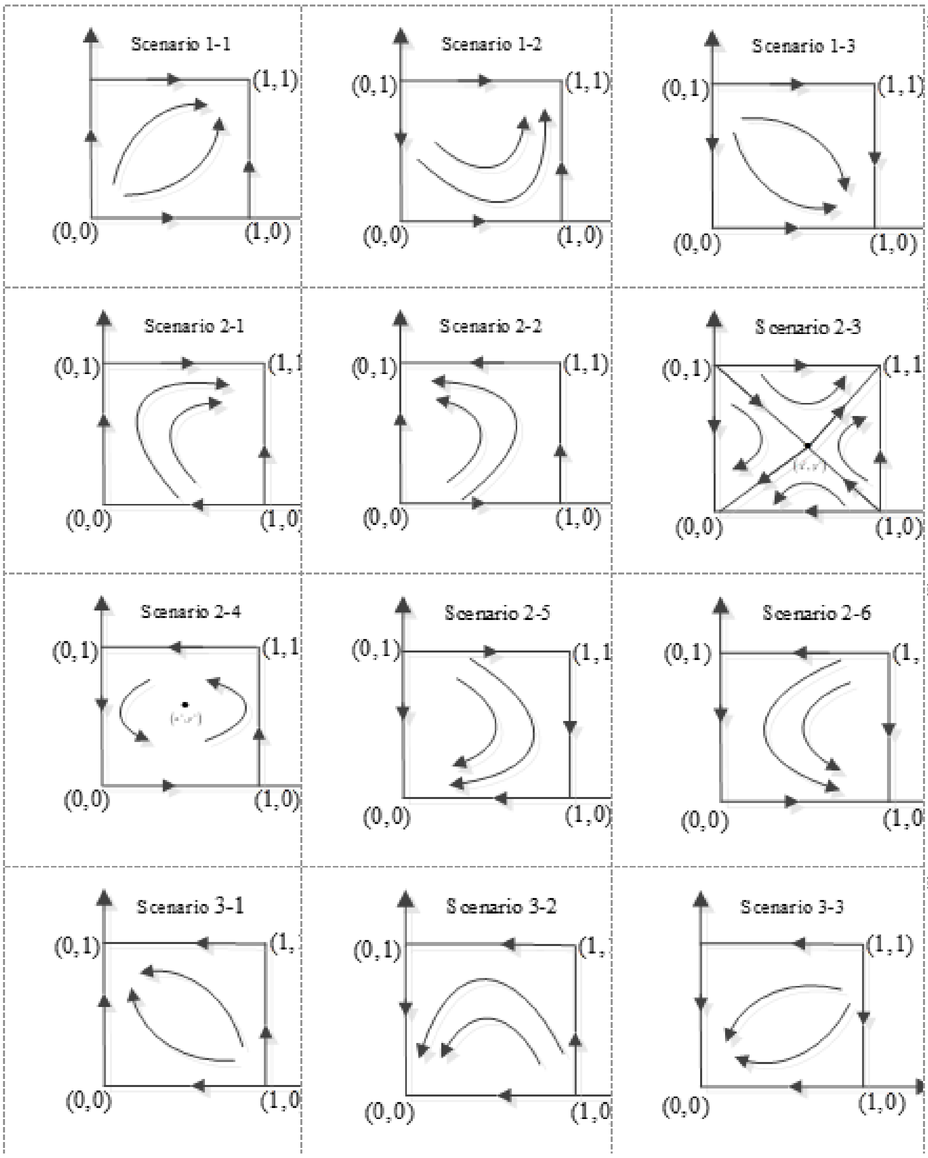


Figure 2. Phase diagram of system evolution in scenarios 1-1 to 3-3

In summary, under scenarios 2-5, 3-2, and 3-3, the system (*) converges to the worst equilibrium E (0,0), indicating that TEPG practices are evolving towards extreme deterioration and are destined to fail in the long run. To mitigate the moral hazards of both PEs and TESEs and build trust relationships for active collaboration, the relationships among the costs of third-party governance, profits of PEs, and associated profit-loss values of environmental management must be coordinated through three mechanisms: government reward-penalty policies, flexible market pricing mechanisms, and mutual supervision parameters. Once $\Delta(B_1+\beta_1)\phi_d + f_{TE}\eta_{TE}(P_0-\Delta\phi_N) > C_P - C_N$ and $(\beta_2-B_1)(\Delta-1)\phi_P-(B_1-1)(\Delta-1)\phi_N > R(P_0+P_1)-R(P_0)-(f_{PE}\eta_{PE}+\beta_2)P_1$ hold, the system's evolutionary equilibrium path will increasingly align with optimal equilibrium ESS (1,1) , that is, scenarios 1-1, 1-2, and 2-1. However, two key considerations require attention during parameter regulation. First, prevent inappropriate parameter adjustments from leading to isolated operations where only one party engages in environmental management as observed ESS (1,0) in scenarios 1-3 and 2-6, and ESS (0,1) in scenarios 2-2 and 3-1. Second, due to the complexity and multiplicity of evolutionary paths, while ensuring optimal equilibrium conditions, particular attention should be paid to the impact of initial system states on final trajectories—especially the coexistence of ESS (0, 0) and ESS (1,1) in scenario 2-3.

Table 4. Stability analysis of equilibrium points in scenarios 2-1, 2-2, and 2-3

equilibrium points	$\min\{(B_1+\beta_1)\phi_d+f_{TE}\eta_{TE}(P_0-\phi_N),\Delta(B_1+\beta_1)\phi_d+f_{TE}\eta_{TE}(P_0-\Delta\phi_N)\}<C_P-C_N<\max\{(B_1+\beta_1)\phi_d+f_{TE}\eta_{TE}(P_0-\phi_N),\Delta(B_1+\beta_1)\phi_d+f_{TE}\eta_{TE}(P_0-\Delta\phi_N)\}$																	
	$R(P_0+P_1)-R(P_0)-(f_{PE}\eta_{PE}+\beta_2)P_1<(\beta_2-f_{TE}\eta_{TE}-B_1)(\Delta-1)\phi_N$						$(\beta_2-f_{TE}\eta_{TE}-B_1)(\Delta-1)\phi_N<R(P_0+P_1)-R(P_0)-(f_{PE}\eta_{PE}+\beta_2)P_1<(\beta_2-B_1)(\Delta-1)\phi_N+(\beta_2-1)(\Delta-1)\phi_d$						$R(P_0+P_1)-R(P_0)-(f_{PE}\eta_{PE}+\beta_2)P_1>(\beta_2-B_1)(\Delta-1)\phi_N+(\beta_2-1)(\Delta-1)\phi_d$					
	$(B_1+\beta_1)\phi_d>f_{TE}\eta_{TE}\phi_N$			$(B_1+\beta_1)\phi_d<f_{TE}\eta_{TE}\phi_N$			$(B_1+\beta_1)\phi_d>f_{TE}\eta_{TE}\phi_N$			$(B_1+\beta_1)\phi_d<f_{TE}\eta_{TE}\phi_N$			$(B_1+\beta_1)\phi_d>f_{TE}\eta_{TE}\phi_N$			$(B_1+\beta_1)\phi_d<f_{TE}\eta_{TE}\phi_N$		
	Scenario 2-1			Scenario 2-2			Scenario 2-3			Scenario 2-4			Scenario 2-5			Scenario 2-6		
	det *J*	tr*J*	stability	det *J*	tr*J*	stability	det *J*	tr*J*	stability	det *J*	tr*J*	stability	det *J*	tr*J*	stability	det *J*	tr*J*	stability
$(0,0)$	$+$	$+$	unstable	$+$	$+$	unstable	$+$	$-$	*ESS*	$-$	?	saddle point	$+$	$-$	*ESS*	$-$	?	saddle point
$(0,1)$	$-$	?	saddle point	$+$	$-$	*ESS*	$+$	$+$	unstable	$-$	?	saddle point	$+$	$+$	unstable	$-$	?	saddle point
$(1,0)$	$-$	?	saddle point	$-$	?	saddle point	$+$	$+$	unstable	$-$	?	saddle point	$-$	?	saddle point	$+$	$-$	*ESS*
$(1,1)$	$+$	$-$	*ESS*	$-$	?	saddle point	$+$	$-$	*ESS*	$-$	?	saddle point	$-$	?	saddle point	$+$	$+$	unstable
(x^*,y^*)	the point is meaningless under this condition			the point is meaningless under this condition			$-$	?	saddle point	$+$	0	center point	the point is meaningless under this condition			the point is meaningless under this condition		

Table 5. Stability analysis of equilibrium points in scenarios 3-1, 3-2, and 3-3

equilibrium points	$C_P-C_N>\max\{(B_1+\beta_1)\phi_d+f_{TE}\eta_{TE}(P_0-\phi_N),\Delta(B_1+\beta_1)\phi_d+f_{TE}\eta_{TE}(P_0-\Delta\phi_N)\}$								
	$R(P_0+P_1)-R(P_0)-(f_{PE}\eta_{PE}+\beta_2)P_1<(\beta_2-f_{TE}\eta_{TE}-B_1)(\Delta-1)\phi_N$			$(\beta_2-f_{TE}\eta_{TE}-B_1)(\Delta-1)\phi_N<R(P_0+P_1)-R(P_0)-(f_{PE}\eta_{PE}+\beta_2)P_1<(\beta_2-B_1)(\Delta-1)\phi_N+(\beta_2-1)(\Delta-1)\phi_d$			$R(P_0+P_1)-R(P_0)-(f_{PE}\eta_{PE}+\beta_2)P_1>(\beta_2-B_1)(\Delta-1)\phi_N+(\beta_2-1)(\Delta-1)\phi_d$		
	Scenario 3-1			Scenario 3-2			Scenario 3-3		
	det *J*	tr*J*	stability	det *J*	tr*J*	stability	det *J*	tr*J*	stability
$(0,0)$	$-$	?	saddle point	$+$	$-$	*ESS*	$+$	$-$	*ESS*
$(0,1)$	$+$	$-$	*ESS*	$-$	?	saddle point	$-$	?	saddle point
$(1,0)$	$+$	$+$	unstable	$+$	$+$	unstable	$-$	?	saddle point
$(1,1)$	$-$	?	saddle point	$-$	?	saddle point	$+$	$+$	unstable
(x^*,y^*)	the point is meaningless under this condition			the point is meaningless under this condition			the point is meaningless under this condition		

3.3. Boundary conditions for optimizing the third-party market technical environment

As analyzed in Section 3.2, $\Delta(B_1+\beta_1)\phi_d + f_{TE}\eta_{TE}(P_0-\Delta\phi_N) > C_P - C_N$ and $(\beta_2-1)(\Delta-1)\phi_P-(B_1-1)(\Delta-1)\phi_N > R(P_0+P_1)-R(P_0)-(f_{PE}\eta_{PE}+\beta_2)P_1$ are satisfied, the system's evolutionary equilibrium path will increasingly align with optimal equilibrium ESS $(1,1)$. However, merely maintaining these conditions to encourage active participation in TEPG is insufficient for establishing a long-term mechanism. The resource-based view suggests that a firm's unique resources and capabilities form the foundation for sustainable development. The technological level of TESEs represents a distinctive competitive advantage in the TEPG market, enabling differentiated profit-generating capabilities. Therefore, this research further explores how to guide TESEs toward advanced technical capabilities in pollution management under market optimal equilibrium conditions, thereby supporting TEPG's sustainable development. Based on the conditions that ESS $(1,1)$ holds, this paper further analyzes how to guide the TEPG market toward a path of sustainable development.

Proposition 3. (1)When $\beta_2>1$, if $\frac{P_P^h-P_P^l}{P_N^h-P_N^l} > \max\{\frac{B_1+\beta_1+f_{TE}\eta_{TE}}{B_1+\beta_1}, \frac{B_1-1}{\beta_2-1}\}$, or (2) When $\frac{(B_1-1)(B_1+\beta_1)}{B_1+\beta_1+f_{TE}\eta_{TE}}+1<\beta_2<1$ and $B_1<1$, if $\frac{B_1+\beta_1+f_{TE}\eta_{TE}}{B_1+\beta_1} < \frac{P_P^h-P_P^l}{P_N^h-P_N^l} < \frac{B_1-1}{\beta_2-1}$, the probability that TESEs pursuing high technology standards is at a heightened level, namely

$$\theta > \max\{\frac{\Delta(B_1+\beta_1+f_{TE}\eta_{TE})P_N^l-\Delta(B_1+\beta_1)P_P^l+C_P-C_N-f_{TE}\eta_{TE}P_0}{\Delta[(B_1+\beta_1)(P_P^h-P_P^l)-(B_1+\beta_1+f_{TE}\eta_{TE})(P_N^h-P_N^l)]}, \frac{R(P_0+P_1)-R(P_0)-(f_{PE}\eta_{PE}+\beta_2)P_1-(\Delta-1)[(\beta_2-1)P_P^l-(B_1-1)P_N^l]}{(\Delta-1)[(\beta_2-1)(P_P^h-P_P^l)-(B_1-1)(P_N^h-P_N^l)]}\}.$$

As can be observed from Proposition 3 (2), this scenario imposes significant restrictions on the range of values for parameter B_1 and β_2, which implies that it corresponds to fewer practical scenarios. Additionally, there is limited flexibility for market and reward-penalty mechanisms to function—only adjustments to parameters to satisfy the condition $\frac{B_1+\beta_1+f_{TE}\eta_{TE}}{B_1+\beta_1} < \frac{P_P^h-P_P^l}{P_N^h-P_N^l} < \frac{B_1-1}{\beta_2-1}$ are feasible. Therefore, this paper does not focus on analyzing this scenario but instead emphasizes the conditions outlined in Proposition 3 (1).

Corollary. During the collaborative pollution treatment, there are the following mechanisms to steer TESEs toward sustainable development with high technology levels according to Proposition 3 (1):

a) When $0<B_1<\frac{1}{2}(\beta_1-\beta_2+\sqrt{(\beta_1-\beta_2)^2+4[\beta_1\beta_2+f_{TE}\eta_{TE}(\beta_2-1)]})$, the TEPG market can be guided towards sustainable development by promoting market flexible pricing B_1, increasing the government's reward-penalty coefficient β_1 for TESEs' governance outcomes, and decreasing the supervision penalties $f_{TE}\eta_{TE}$ to satisfy the condition $\frac{P_P^h-P_P^l}{P_N^h-P_N^l} > \frac{B_1+\beta_1+f_{TE}\eta_{TE}}{B_1+\beta_1}$.

b) When $B_1>\frac{1}{2}(\beta_1-\beta_2+\sqrt{(\beta_1-\beta_2)^2+4[\beta_1\beta_2+f_{TE}\eta_{TE}(\beta_2-1)]})$, the TEPG market can be guided towards sustainable development by lowering market flexible pricing B_1 and increasing the government's reward-penalty coefficient β_2 for the PEs to satisfy the condition $\frac{P_P^h-P_P^l}{P_N^h-P_N^l} > \frac{B_1-1}{\beta_2-1}$. Corollary centers around $\frac{1}{2}(\beta_1-\beta_2+\sqrt{(\beta_1-\beta_2)^2+4[\beta_1\beta_2+f_{TE}\eta_{TE}(\beta_2-1)]})$, adjusting market parameters B_1 to deploy sustainable development paths. Specifically, increasing market flexible pricing B_1 in a) but not exceeding this center value, or reducing it in b) but not going below it, thus aligning the growth needs of the TEPG market across various development phases. Governments' reward-penalty coefficient should be selectively adjusted based on the boundary conditions of market payment changes. In scenario a) where market flexible pricing B_1 is relatively low, the reward-penalty coefficient for TESEs should be increased to compensate their innovation costs for high technological levels. In scenario b) where B_1 is relatively higher than that in a), the reward-penalty coefficient for PEs should be enhanced to motivate contracting high-tech TESEs, using obtained environmental benefits to offset elevated management costs. Mutual supervision mechanisms primarily function in pathway a) by reducing supervision penalties imposed by PEs on TESEs, aiming to alleviate expenditure costs associated with high-level technological innovations. Therefore, whether through adjustment pathway a) or b), ensuring sustainable and high-quality development of the TEPG market requires careful coordination among policy-based reward-penalty, market pricing, and mutual supervision. This framework provides theoretical guidance for steering TEPG practices toward sustainability across various developmental scenarios.

4. Case Study Analysis

Building upon the established model, the following case analysis examines the correctness of the theoretical analysis results and how key factors influence the collaborative governance pathway. Specifically, we investigate the effects of policy-based reward-penalty (namely β_1 and β_2), market pricing (namely payment parameter B_1), mutual supervision (reflected in parameter

η_1 and η_2), and the technical level of TEPG market (reflected in parameter θ). This analysis aims to provide decision support for managers guiding collaborative pollution treatment behavior, demonstrating the practical application of the model.

The case information is sourced from the civil judgment of the Wuxi City Intermediate People's Court in Jiangsu Province, China, as recorded on the China Judgments Online, titled "Second Instance Civil Judgment on the Contract Dispute Between Wuxi Aofeng Wool Washing Co., Ltd. and Jiangsu Qingshuiyuan Environmental Protection Facility Operation Co., Ltd." This case provides a typical example of TEPG partnerships and offers quantifiable data on costs, penalties, and emissions, making it ideal for our simulation. The model parameters are derived and calibrated from the case documents as follows. To simplify computational load without altering the quantitative relationships between parameters, we proportionally scale down the source data. (1) According to the "Wastewater Treatment Commissioning Agreement," Aofeng Company shall pay Qingshuiyuan Company a monthly operational management fee of 35×10^4 yuan for wastewater management. This fee serves as the source data for estimating the active management cost C_P of TESEs, which is subsequently scaled to a model value of C_P=35 yuan. Given the actual operational context $C_P > C_N$, we assume C_N=20 yuan. (2) According to the civil judgment providing information on the wastewater treatment of AoFeng Company, it is known that in January and February 2014, the pollution emissions were 0.1263×10^4 and 0.7182×10^4 tons respectively. These values serve as a reference range for estimating the parameters P_P^h , P_P^l , P_N^h , and P_N^l . Simultaneously, considering the size relationship among these four parameters, they are correspondingly scaled to model values of P_P^h=0.3 , P_P^l=0.2 , P_N^h=0.2 , and P_N^l=0.15 tons. (3) As the "Wastewater Treatment Commissioning Agreement" stipulates that QingShuiYuan Company treats the wastewater discharged by AoFeng Company directly by modifying the treatment facilities to meet standards, thus $P_0 \le \max\{P_P^h, P_P^l, P_N^h, P_N^l\}$, it is proposed here that P_0=0.3 tons. (4) According to Article 4 of the Agreement, if Qingshuiyuan Company breaches the contract, it shall bear a penalty equivalent to 30% of the monthly management fee. This penalty serves as the source data for mutual supervision punishment, leading to the setting of η_{PE}=η_{TE}=100 yuan per ton. (5)Finally, referencing the penalty of 40.9787×10^4 yuan imposed by the Wuxi Municipal, Gardens and Landscaping Bureau, Xishan Environmental Protection Bureau, and Wuxi Xishan District Construction Bureau on AoFeng Company for excessive wastewater discharge, and considering the construction of policy-based reward-penalty coefficient in the model, we set β_1=β_2=50 yuan per ton. Other parameter data are assumed based on their meanings, value constraints, and actual conditions from business consultations, respectively setting $B_1 = 300$ yuan per ton, $\theta = 0.6$, $\Delta = 2$, $P_1 = 0.08$ tons, $R(P_0)$=800 yuan, $R(P_0+P_1)$=1000 yuan.

To guide the environmental protection cooperation pathway, it is essential to consider both the impact of key parameters and the system's initial conditions. Accordingly, we assign different initial values when examining how key parameters influence the evolution of the TEPG game model. Specifically, we investigate how the system evolution (namely $x(t)$ and $y(t)$) varies with the flexible pricing parameter B_1 , which takes values from 200 to 400 in increments of 100. This analysis is conducted for three initial value scenarios: low ($x_0 = 0.1$ and $y_0 = 0.1$), medium ($x_0 = 0.5$ and $y_0 = 0.5$), and high ($x_0 = 0.9$, $y_0 = 0.9$), while keeping other model parameters unchanged as previously defined. The simulation results are shown in **Figure 3**. Similarly, under low, medium, and high initial values, we simulate trajectory changes for: (1) Policy-based reward-penalty parameters β_1 and β_2 taking values from 50 to 100 in increments of 25 (**Figure 4**); (2) Mutual supervision parameters η_1 and η_2 taking values from 100 to 300 in increments of 100 (**Figure 5**); (3) Proportion of high-tech TESEs θ taking values from 0.1 to 0.9 in increments of 0.4 (**Figure 6**).

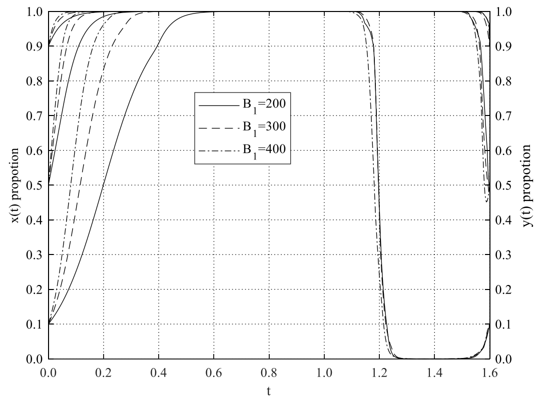


Figure 3. Impact of B_1 on system evolution

Regarding **Figure 3**, it follows that increasing B_1 goes to enhance the system's convergence rate regardless of the initial system values. Furthermore, for a fixed value of B_1, progressively higher initial system values positively impact the convergence speed. As shown in **Figures 4 and 5**, for TESEs, when x_0 = 0.5 and x_0 = 0.9 , the changes of β_1, β_2, η_1, and η_2 are not much pronounced on the direction and speed of system evolution trajectory, with $x(t)$ all quickly trending to 1. When x_0 = 0.1, raising β_1, β_2, η_1, and η_2 goes to effectively speeds up the convergence of $x(t)$ to 1. For a fixed value of β_1, β_2, η_1, and η_2, when x_0 increase from 0.1 to 0.9, the convergence speed of $x(t)$ to 1 is also faster. These results indicate that when TESEs' endogenous motivation is already high, external interventions (policy and supervision parameters) have limited effect on promoting proactive governance strategies. However, when endogenous motivation is low, external guidance measures (e.g., increasing flexible pricing and strengthening policy support) play significant roles in facilitating the ideal equilibrium.

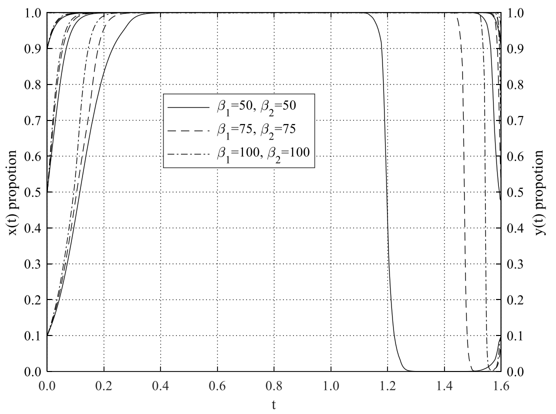


Figure 4. Impact of β_1 and β_2 on system evolution

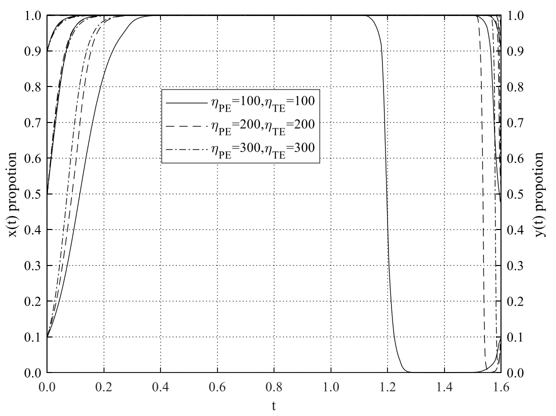


Figure 5. Impact of η_1 and η_2 on system evolution

For PEs, increasing β_1, β_1, η_1, and η_2 accelerates the convergence of $y(t)$ to 1, notably so when y_0 =0.1, where the enhancing effect is more pronounced in **Figures 4 and 5**. Promoting flexible market prices B_1 lowers the strategic benefits for PEs to adhere to compliance, thus posing an obstacle to the convergence of $y(t)$ 1. However, in our case study, this hindrance is not substantial. Instead, the system's endogenous initial values have a more significant impact on the stable KP strategy choices of PEs, causing $y(t)$ rapidly approach 1. Section 3.3 explores guidance mechanisms for developing high-tech TEPG markets based on collaborative environmental governance pathways, thereby creating a virtuous cycle of sustainable environmental governance. Conversely, high-tech markets also promote positive environmental behaviors among participants. This perspective is fully illustrated in **Figure 6**, particularly at $y_0 = 0.1$, where the greater the θ will markedly create a shorter time to the stationary state. These results adequately reflect the complementary and interactive relationship between the technical development of TEPG markets and their collaborative governance pathways.

Using the parameter settings established previously, this study further adjusts the parameter P_P^h=P_0=0.21 to satisfy the condition $\frac{P_P^h - P_P^l}{P_N^h - P_N^l} > \frac{B_1 - 1}{\beta_2 - 1}$ in Corollary b. The results demonstrate that managers can guide TESEs toward sustainable development paths when θ exceeds a certain threshold (Based on Proposition 3 and our simulation parameters, this threshold is determined as $\frac{\Delta(B_1+\beta_1+f_{TE}\eta_{TE})P_N^l - \Delta(B_1+\beta_1)P_P^l + C_P - C_N - f_{TE}\eta_{TE}P_0}{\Delta(B_1+\beta_1)(P_P^h - P_P^l) - \Delta(B_1+\beta_1+f_{TE}\eta_{TE})(P_N^h - P_N^l)}$). Given that this threshold determines the minimum required proportion of high-tech TESEs, we further examine how simultaneous adjustment of B_1 and β_1 affects this threshold (**Figure 7**). **Figure 7** clearly demonstrates that increasing B_1 has a significant positive effect on this threshold, while the effect of β_2 is less pronounced. This indicates that the scaling potential of high-tech TESEs is ultimately market-driven. Furthermore, we investigate threshold variations under combined influences of B_1 and β_1 (**Figure 8**), and B_1 and η_{TE} (**Figure 9**). Results clearly show that threshold elevation depends primarily on higher market prices B_1, while increasing β_1 or decreasing η_{TE} can reduce investment costs for high-tech pollution treatment, thereby facilitating growth in the proportion of high-tech TESEs.

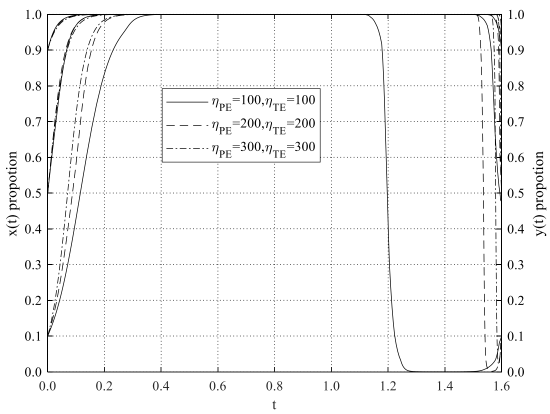


Figure 5. Impact of η_1 and η_2 on system evolution

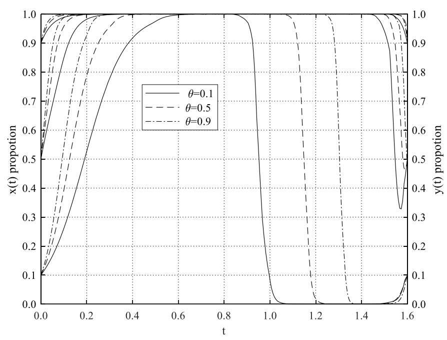


Figure 6. Impact of θ on system evolution

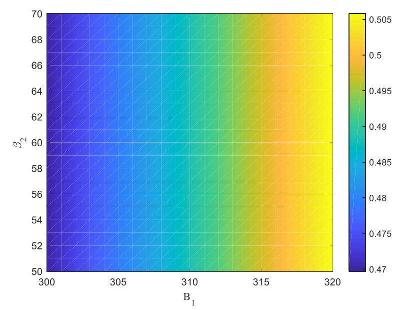


Figure 7. The threshold changes under the simultaneous influence of B_1 and β_2

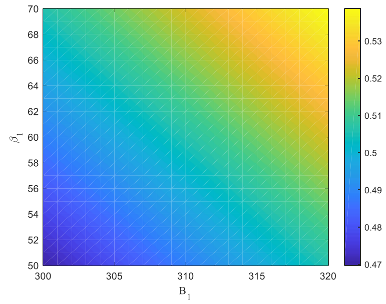


Figure 8. The threshold changes under the simultaneous influence of B_1 and β_1

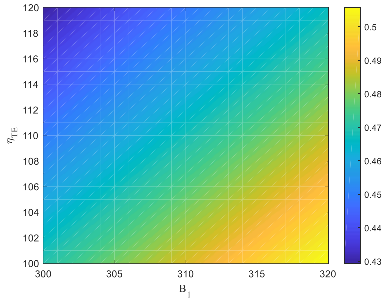


Figure 9. The threshold changes under the simultaneous influence of B_1 and η_{TE}

5. Conclusions

Anchored in TEPG practice, this study begins by distinguishing two potential categories of TESEs to account for heterogeneity in their technological levels. An evolutionary game model is then formulated to describe collaborative governance between PEs and TESEs, incorporating government reward-penalty policy parameters, market-based flexible pricing mechanisms, and mutual supervision parameters. Building on this foundation, the paper analyzes the optimal evolutionary trajectory and the mechanism for improving the scale of high-tech TESEs. Finally, a case study is utilized to examine how critical parameters foster collaborative environmental governance and enhance the technological landscape of the TEPG market. The research findings demonstrate that: (1) The TEPG evolutionary game system contains an optimal equilibrium point (1, 1) where TESEs actively manage pollution and PEs honor discharge contracts; suboptimal equilibria (0, 1) or (1, 0) where one participant deviates from cooperation; and the worst equilibrium point (0, 0) with no cooperation. (2) Realizing collaborative governance pathways requires coordinated application of policy, market, and supervision parameters to adjust profit-loss values, achieving conditions specified in Scenarios 1-1, 1-2, and 2-1 (Proposition 2). (3) Developing high-tech pollution treatment requires identifying differential market mechanism effects under varying conditions, supplemented by policy and supervision measures meeting Proposition 3 conditions, thus steering the market toward sustainable, high-quality development. (4) When participants exhibit high endogenous motivation for collaborative pollution control, the system rapidly converges to optimal stability. With low motivation, intensifying government reward-penalty measures, increasing mutual supervision penalties, appropriately setting flexible market prices, and expanding high-tech TESEs can effectively accelerate collaborative governance dominance.

In fact, beyond TESEs and PEs as primary participants, other entities, including government agencies, the public, and media, also influence collaborative environmental governance directions in TEPG markets. Thus, evolutionary mechanisms and guidance measures involving multiple stakeholders represent important avenues for future research. Additionally, while this study simplifies TESEs into high-tech and low-tech categories, technological parameters are mostly continuous values in reality. Future research could develop continuous or functional technological parameters based on field investigations to examine collaborative environmental governance pathways within evolutionary game frameworks.

References

Cao T. Y., Sun F. Q. and Sun T. (2023), Spatial test of collaborative treatment efficiency measurement and driving factors of air pollution in China's Yangtze River Delta urban agglomeration, *Urban Climate*, 49, 101454.

Chen X. H., Cai S. J. and Wang Y. J. (2020), The institutional and policy logic of the implementation of the environmental protection supervision system in China, *Journal of Management World*, 36(11), 160-172.

Chen X. H., Yu Z. M. and Li J. L. (2019), Outsourcing contract for pollution abatement service with risk preference consideration, *Control and Decision*, 34(12), 2690-2697.

Chien S. S. and Hong D. L. (2018), River leaders in China: Party-state hierarchy and transboundary governance, *Political Geography*, 62, 58-67.

Chong D. S. and Sun N. (2020), Explore emission reduction strategy and evolutionary mechanism under central environmental protection inspection system for multi-agent based on evolutionary game theory, *Computer Communications*, 156, 77-90.

Chong D. S. and Sun N. (2020), Explore emission reduction strategy and evolutionary mechanism under central environmental protection inspection system for multi-agent based on evolutionary game theory, *Computer Communications*, 156, 77-90.

Dai J., Yan J. and Gao H. (2022), Study of Chinese carbon emission trading market mechanism based on the game theory, *Global NEST Journal*, 24(1), 9-15.

Du J. G., Zhao L. and Chen L. (2015), Study on the evolution game between the governments and the third party pollution treatment companies, *Science and Technology Management Research*, 35(14), 214-218.

Friedman D. (1991), Evolutionary games in economics, *Econometrica*, 59(3), 637-666.

Gao X. W. and Hua Y. J. (2022), Environmental Protection Tax Law on the synergy of pollution reduction and carbon reduction in China: Evidence from a panel data of 107 cities, *Sustainable Production and Consumption*, 33, 425-437.

Guo J. X., Lin O. W., Ye K. Y. Guan J B and Lei X. (2024), Research on regulatory strategies of green finance and optimization of corporate green behavior under the framework of circular economy, *Global NEST Journal*, 26(4), 05913.

Guo L. B., Zhang Q. Q., Wu J. and Gonzalez E. S. (2024), An evolutionary game model of manufacturers and consumers' behavior strategies for green technology and government subsidy in supply chain platform, *Computers & Industrial Engineering*, 189, 109918.

Hosseini-Motlagh S. M., Choi T. M., Johari M. and Nouri-Harzvili M. (2022), A profit surplus distribution mechanism for supply chain coordination: An evolutionary game-theoretic analysis, *European Journal of Operational Research*, 301, 561-575.

Jia F. J., Wang D. D., Zhou K. and Li L S. (2021), Differential decision analysis of transboundary pollution considering the participation of the central government, *Managerial and Decision Economics*, 43(6), 1684-1703.

Jiao Y. Q., Su M. R., Ji C. W., Yang S. and Zhang P. (2021), How to design fully cooperative policies to abate transboundary air pollution between two highly asymmetric regions: An abnormal incrementalism analysis, *Journal of Cleaner Production*, 278, 124042.

Kim S. K., Gevelt T. V., Joosse P. and Bennett M. (2022), Transboundary air pollution and cross-border cooperation: Insights frommarine vessel emissions regulations in Hong Kong and Shenzhen, *Sustainable Cities and Society*, 80, 103774.

Klenert D., Mattauch L., Combet E., Edenhofer O., Hepburn C., Rafaty R. and Stern N. (2018), Making carbon pricing work for citizens, *Nature Climate Change*, 8(8), 669-677.

Kong L. Q., Li Z. Y., Liu B. Q., Liu B. Q. and Xu K. (2024), The impact of environmental protection tax reform on low-carbon total factor productivity: Evidence from China's fee-to-tax reform, *Energy*, 290, 130216.

Lei X., Lin O. W. and Lin J. J. (2024), Evolutionary game analysis of enterprises' green production behavior in the context of china's economic green transformation, *Global NEST Journal*, 26(3), 05781.

Li H. Q. and Guo G. L. (2019), A differential game analysis of multipollutant transboundary pollution in river basin. *Physica A*, 535, 122484.

Li Z. and Li Y. Q. (2024), Environmental regulation and employment: Evidence from China's new environmental protection law, *Economic Analysis and Policy*, 82, 400-416.

Liu C. (2015), Regulation, interaction and environmental pollution control by a third party, *China Population, Resources and Environment*, 25(2), 96-104.

Liu D. H., Xiao X. Z., Li H. Y. and Wang W. (2015), Historical evolution and benefit-cost explanation of periodical fluctuation in coal mine safety supervision: An evolutionary game analysis framework, *European Journal of Operational Research,* 243, 974-984.

Lu J. (2022), Can the central environmental protection inspection reduce transboundary pollution? Evidence from river water quality data in China, *Journal of Cleaner Production*, 332, 130030.

Lu Y. L., Wang Y., Zhang W., Hubacek K., Bi F., Zuo J., Jiang H., Zhang Z., Feng K. and Liu Y. (2019), Provincial air pollution responsibility and environmental tax of China based on interregional linkage indicators, *Journal of Cleaner Production*, 235, 337-347.

Meng C. S., Tang Q., Yang Z. H., Cheng H. and Li K. (2021), Collaborative control of air pollution in the Beijing-Tianjin-Hebei region, *Environmental Technology & Innovation*, 23, 101557.

Ren Z. R. (2021), The third Party Control of Environmental pollution: The types of disputes and the approach of Judicial Relief, *Theory Journal*, 6, 111-121.

Sun H. X. and Li J. L. (2021), Behavioural choice of governments, enterprises and consumers on recyclable green logistics packaging, *Sustainable Production and Consumption*, 28, 459-471.

Tang S. J. and Wei Y. (2020), On the definition of environmental tort liability under the invalidity of third-party governance contract, *Journal of Chongqing University (Social Science Edition)*, 5, 177-187.

Tian M., Xu G. H. and Zhang L. Z. (2019), Does environmental inspection led by central government undermine Chinese heavy-polluting firms' stock value? The buffer role of political connection, *Journal of Cleaner Production*, 236, 117695.

Wang W., Wu F. P. and Yu H. (2022), Optimal design of the ecological compensation mechanism in transboundary river basins under the Belt and Road Initiative, *Sustainable Production and Consumption*, 32, 173-183.

Xing Z. C. (2023), Collaborative governance of trade-driven transboundary air pollution in China: A responsibility-based fair compensation mechanism, *Journal of Environmental Management*, 348, 119327.

Xu R., Wang Y. R., Wang W. B. and Ding Y. (2019), Evolutionary game analysis for third-party governance of environmental pollution, *Journal of Ambient Intelligence and Humanized Computing*, 10, 3143-3154.

Yang G. Q., Zhang Y. S., Wang H. S. and Chen M. (2024), The environmental and economic utility of the third-party governance of environmental pollution in China–an analysis based on the business registration information of enterprises, *Journal of Environmental Planning and Management*, 67(14), 3502-3527.

Yang Y., Zhao L. J., Wang C. C. and Xue J. (2021), Towards more effective air pollution governance strategies in China: A systematic review of the literature, Journal of Cleaner Production, 297, 126724.

Yang Y., Zhao L. J., Wang C. C. and Xue J. (2021), Towards more effective air pollution governance strategies in China: A systematic review of the literature, *Journal of Cleaner Production*, 297, 126724.

Yang Z., Niu G. M. and Lan Z. R. (2021), Policy strategy of transboundary water pollution control in boundary rivers based on the evolutionary game, *China Environmental Science*, 41(11), 5446-5456.

Zhang B., Chen X. L. and Guo H. X. (2018), Does central supervision enhance local environmental enforcement? Quasi-experimental evidence from China, *Journal of Public Economics*, 164, 70-90.

Zhang G. X., Ye Y. Q., Guan X., Yin J. H. and Lv X L. (2018), Difference and collaboration in Jing-Jin-Ji's energy saving and emission reduction policy measurers, *Journal of Management Sciences in China*, 21(5), 111-126.

Zhang Z. H., Ling D., Yang Q. X., Feng Y. C. and Xiu J. (2024), Central environmental protection inspection and carbon emission reduction: A tripartite evolutionary game model from the perspective of carbon neutrality, *Petroleum Science*, 21(3), 2139-2153.

Zhao F. F., Shu X. Y., Zhao X. and Guo M. (2024), Determinants and action paths of transboundary water pollution collaborative governance: A case study of the Yangtze River Basin, China, *Journal of Environmental Management*, 360, 121217.

Zheng J. J., Dong J. H. and Ren T. Y. (2021), A stochastic differential cooperative game based on the third-party governance of environmental pollution, *Journal of Management Sciences in China*, 24(7), 76-93.

Zhou W. J., Chen L. and Feng L. L. (2019). Effectiveness evaluation on third-party governance model for environmental pollution in China, *Environmental Science and Pollution Research*, 26, 17305-17320.

Zhou X., Zhao R., Cheng L. and Min X. (2019), Impact of policy incentives on electric vehicles development: A system dynamics-based evolutionary game theoretical analysis, *Clean Technologies and Environmental Policy*, 21(5), 1039-1053.