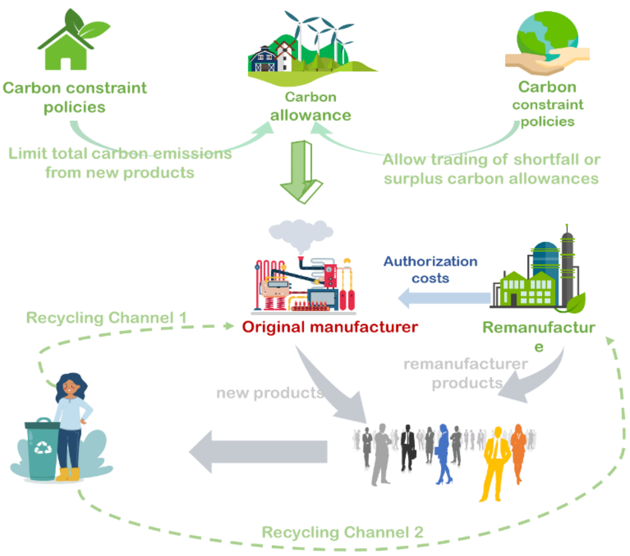
Global NEST Journal, Vol 27, No 6, 07418

Study on the selection of recycling channels for authorized remanufacturing models under carbon allowance

Zishan Cui and Jiuhe Wang*
School of economics and management, Yanshan University, Qinhuangdao 066000, China
Received: 04/03/2025, Accepted: 30/05/2025, Available online: 04/06/2025
*to whom all correspondence should be addressed: e-mail: wjh@ysu.edu.cn
https://doi.org/10.30955/gnj.07418

Graphical abstract



Abstract

To analyze the impact of allocation methods of carbon allowances on the choice of recycling channels for original manufacturers under the authorized remanufacturing model. A Stackelberg game model of original manufacturer recycling and remanufacturer recycling under the grandfathering rule and benchmarking rule is established, which assumes that both decision-making parties are completely rational, and the impacts of different allocation methods of carbon allowances on new and remanufactured products under different recycling channels are investigated. Further, this paper analyzes the impact of consumer green preference and recycling scale on remanufactured products by using numerical simulation with reference to the parameter settings of related literature. The study found that: (1) grandfathering rule are ineffective in promoting original equipment manufacturers' investment in carbon-reducing technologies. Original equipment manufacturers (OEMs) were more likely to respond to government-set grandfathering rule with strategies to increase product pricing or reduce production; (2) Different carbon allowance allocation methods formulated by the government will not directly affect the original manufacturers' licensing fee decisions, but they will affect the original manufacturers' licensing fee decisions by indirectly acting on consumers' green consumption preferences. (3) When the government formulates the grandfathering rule, the original manufacturer will often choose the remanufacturer recycling channel, thus achieving a win-win situation for the interests of both parties.

Keywords: Carbon allowance, authorized remanufacturing, recycling channels, stackelberg game

1. Introduction

With the aggravation of climate change, the call for reducing carbon emissions and combating climate change has been increasing (Zhou and Shan, 2023). Many countries are actively implementing carbon emission reduction programs, carbon emission taxes, carbon trading, and quota systems to achieve this common goal (Li *et al*., 2023). Led by the goal of 'double carbon', China actively promotes the carbon quota policy, carbon quota system becomes an important market-based instrument for manufacturing enterprises to promote green transformation. As a market-based carbon emission reduction policy tool (Liu *et al*., 2024), the carbon quota system incentivizes enterprises to optimize resource allocation and reduce carbon emissions during production and operation by limiting the total amount of carbon emissions and allowing trading of quotas (Xia *et al*., 2024). Currently, two free allocation methods are mainly used: the grandfathering rule and the benchmarking rule for the industry (Ji *et al*., 2017). The grandfathering rule is based on the past years' emission data, and the industry benchmarking rule accounts for the overall emission level of the industry.

Under the influence of carbon quota system, manufacturing enterprises not only need to carry out low-carbon technological innovation on the production side, but also pay more and more attention to the green management of the whole life cycle of products (Selvanarayanan *et al*., 2024). As an important support for the green transformation of enterprises, the recycling channel is not only the main way to obtain waste resources (Hong *et al*., 2024), but also an important means of eliminating the shortage of resources in the solid waste segment and promoting the development of a circular economy (Wu *et al*., 2025) Remanufacturing is gradually becoming an important pathway due to its significant resource-saving and carbon emission benefits (Yu, 2024). The original manufacturer often licenses the right to produce and sell the remanufactured product to the remanufacturer by charging a patent license fee to a third-party independent remanufacturer, which is known as the licensed remanufacturing model. This model has been successfully applied in several cases in the industry (Zhou *et al*., 2020), such as the Volkswagen Group commissioning Volkswagen FAW Engine (Dalian) Co., Ltd (VWED) to produce remanufactured engines by charging a certain amount of patent licensing fees, while the sales are handled by the remanufacturer (VWED). However, the recycling efficiency and quality of used products directly determine the cost and carbon emission benefits of remanufactured products, and the choice of recycling channels is the key to the operational effectiveness of the authorized remanufacturing model.

Specifically, introducing carbon quotas has significantly affected firms' choice of recycling channels. On the one hand, the formulation of different carbon allowance allocation mechanisms is directly related to the carbon cost burden of enterprises (Wang *et al*., 2019); on the other hand, the interest game between the original manufacturer and the remanufacturer in the construction of recycling channels also affects the implementation effect of the low-carbon strategy of enterprises (Kadeer *et al*., 2024). Therefore, firms face multiple trade-offs between recycling scale, quality control, and carbon responsibility. In addition, in high-carbon emitting industries, carbon emission constraints further exacerbate this trade-off, making the rational choice of recycling channels a central challenge in optimizing resource allocation and achieving carbon emission targets.

In summary, the carbon quota allocation method puts forward higher requirements for the selection of recycling channels under the authorized remanufacturing mode. Considering the characteristics of the authorized remanufacturing mode, and in order to promote the rapid development of the remanufacturing industry, this paper considers three key questions: (1) How can manufacturers optimize the authorization fee and the pricing of new products in order to balance the market competition and profit maximization under the constraints of carbon quota? (2) How do different carbon quota allocation methods affect the choice of recycling channels and competition in the manufacturing/remanufacturing market? (3) How can the government and enterprises develop the most effective decision-making model that balances economic and environmental benefits to achieve mutual benefits?

2. Literature review

At present, domestic and foreign remanufacturing research focuses on different aspects. After sorting out the literature, the current research hotspots focus on two aspects: firstly, the choice of remanufacturing recycling channels, and secondly, the impact of carbon quota allocation methods on remanufacturing.

In the context of the study on the selection of recycling channels, focusing on recycling channel selection, Savaskan *et al*. (2004) investigated a closed-loop supply chain structure, including manufacturers and retailers, using a game theoretic approach. Analyses show that the cost of recycling in the retailer recycling channel is lower than in the manufacturer or third-party recycler recycling channel. Based on this, Savaskan and Van Wassenhove (2006) studied the selection of recycling channels in the presence of one manufacturer and two retailers. And then, with the emergence of the carbon emission problem, more and more scholars have researched the choice of recycling channels in different contexts. For example, Huang *et al*. (2017) conducted a study on the existence of dual recycling channels in a closed-loop supply chain and found that the intensity of competition in the recycling channel directly affects recycling costs, both from the perspective of the manufacturer and the consumer. Lu and Li (2016) developed a recycling model in a retailer's competitive environment considering electronic products' life cycle and demand pricing characteristics. Studies have shown that manufacturers choose retailers to recycle when the government determines the recycling rate and recycle better when recyclers determine the recycling rate. Kushwaha *et al*. (2022) studied the channel mix of recycling channels chosen by manufacturers to maximize profits over a limited planning horizon. In addition, Yang *et al*. (2023) have suggested that manufacturers can outsource recycling activities to retailers to form a more effective price incentive mechanism. On the other hand, research on the selection of recycling channels for used power batteries has also attracted extensive attention in recent years. Relevant studies mainly focus on the technological innovation path for high-value recycling of battery materials (Gu *et al*., 2024; Quan *et al*., 2024), as well as the operational efficiency and environmental performance under different recycling modes in the closed-loop supply chain (Jiao *et al*., 2023). Most of the above studies on recycling channel selection ignored the selection of recycling channels under carbon tax policy implementation.

In terms of research on the impact of carbon quota allocation methods on remanufacturing, numerous studies have shown that carbon allowances significantly influence key decisions on remanufacturing activities by constraining and incentivizing firms' carbon emissions (Xia *et al*., 2023). On the one hand, carbon allowances are beneficial for remanufacturing in both ordinary and green markets. They can mitigate the negative impacts of total carbon emission control and carbon trading mechanisms (Chai *et al*., 2018). On the other hand, the constraint of carbon allowances can achieve more favorable production and management strategies (Shu *et al*., 2017). Then, with the establishment and development of the carbon trading market, scholars exam attention to the enterprise production pricing strategy and carbon emission reduction behavior under the carbon trading mechanism, and the research began to explore the impact of carbon trading price fluctuations on the cost-benefit and emission reduction incentives of the enterprise, as well as the reasonable formulation of the government's carbon price (Zhu *et al*., 2024). In recent years, with the promotion of the remanufacturing model, studies have begun to incorporate the relevant contexts. Xia *et al*. (2024) explored the issue of the government's carbon quota allocation method and the choice of remanufacturing model for original remanufacturers in the context of intellectual property protection. They found that original manufacturers would choose different remanufacturing models under different carbon quota allocation methods.

In addition, most of the previous studies adopt the traditional Gono game(Hu *et al*.) or complete information game(Ghosh *et al*., 2020; Huang *et al*., 2021) to analyze, however, these research methods fail to adequately portray the display characteristics of the power difference and decision-making sequence between manufacturers and remanufacturers(Xia *et al*., 2023). In particular, under the constraints of carbon quota policy, manufacturers, as authorized parties, usually have stronger dominant power and pricing first-mover advantage, a feature that has not yet been effectively reflected in traditional approaches. In contrast, the Stackelberg game, as a typical leader-follower game framework(Li *et al*., 2024), can better portray the master-slave relationship between manufacturers and remanufacturers in the context of authorized remanufacturing, and reasonably reflect the decision-making sequence of different subjects and the characteristics of the interest game(Xia *et al*., 2025). Therefore, this paper constructs a decision-making model of recycling channel selection based on the Stackelberg game, which not only makes up for the shortcomings of the existing research in the application of the method, but also fits the actual operation of the authorized remanufacturing situation better, and has stronger theoretical value and practical guiding significance.

Summarizing the above literature, it is found that the literature on recycling channel selection considers the impact of different recycling channels and competitive intensity on recycling costs and recycling channel selection. A study on the impact of carbon quota allocation methods on remanufacturing only considered the impact of carbon quota policy on the operational decision-making of remanufacturing in their studies while ignoring the recycling channel selection, which is a key aspect of remanufacturing. In summary, this paper concludes that the problems and shortcomings of the above research content: (1) the existing research ignores the integrated impact of carbon quota and different recycling channels on the recycling cost and the decision-making process of channel selection; (2) there are fewer studies on what kind of systematic stability strategy combinations of the carbon quota trading mechanism and recycling channels have, in particular, the existing literature seldom takes into account the win-win situation of economic and environmental benefits; (3) fewer literature integrates the context of intellectual property protection, especially in the process of recycling waste products.

3. Model description and analysis

3.1. Description of the model

In this paper, we construct a remanufacturing game model consisting of an original manufacturer and a remanufacturer, and Xia *et al*. (2023) regards the original manufacturer as the dominant player in the game and the remanufacturer as the follower. Original manufacturers, which are responsible for the production and sale of new products protected by patents and the setting up of recycling channels for used products, are high-carbon-emitting enterprises regulated by the government's carbon tax policy. Under the carbon trading policy, the government sets the carbon trading price and determines the initial carbon emission quotas based on the grandfathering rule and the benchmarking rule, respectively. Owing to technological and financial constraints on remanufacturing, OEMs choose recycling channels to collect used products and license remanufacturing production and sales operations to remanufacturers. Considering the protection of intellectual property rights, the OEMs charge a licensing fee so that the remanufactured products compete with new products in the market. Taking the Volkswagen Group as an example, the Volkswagen Group authorized the remanufacturing right of the engine EA888 by charging a licensing fee to Volkswagen FAW (Dalian) Co. In this paper, the manufacturing/remanufacturing game models under two recycling channels are constructed based on the different allocation methods of carbon quotas set by the government, respectively, as shown in **Figure 1**.

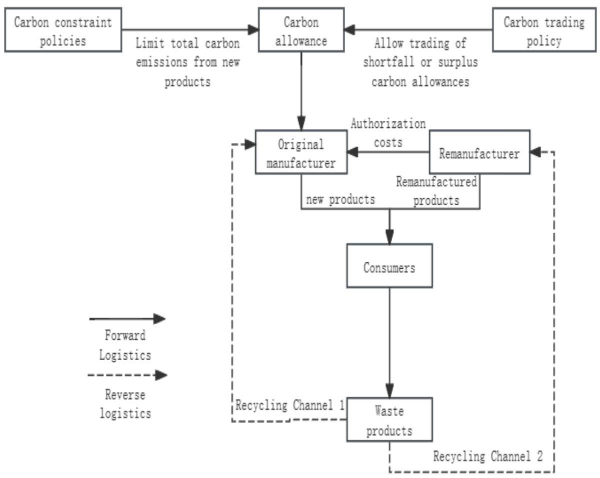


Figure 1. Original Manufacturer Recovery Channel Manufacturing/Remanufacturing Game Models

3.2. Description of symbols

The symbols and descriptions used in this paper are specified in **Table 1**.

Table 1. Description of symbols

notation	description
decision variables	
p_{in}^j, p_{inr}	Denote the unit pricing of new and remanufactured products in the two recycling channels under mode j, respectively, where j∈{L,H},i∈{OR,RR}
τ_i^j	denotes the recovery rate of used products from the two recycling channels under model j
Z_i^j	Indicates the cost per unit of remanufacturing authorization for both recycling channels under model j
parameters	
OR, RR	Indicates original manufacturer recycling channel and remanufacturer recycling channel, respectively
L, H	Indicates historical emission method and sectoral baseline method, respectively
n, r	Indicates original manufacturer and remanufacturer, respectively
k	Indicates the scale factor for recycling used products, the larger the value, the greater the recycling cost for recycling the same amount of used products
E, α	Indicates the total amount of carbon allowances determined by the Government on the basis of the historical emissions method and the baseline carbon allowances determined on the basis of the baseline method, respectively.
q_{in}^j, q_{ir}^j	Denote the sales of new and remanufactured products from the two recycling channels under mode j, respectively
e_n, p_e	Indicates the carbon emissions per unit and the carbon trading price per unit of the new product, respectively
c_n, c_r	Representation of production uplift costs for new and remanufactured products, respectively; $c_n>c_r$
δ	Indicates the green preference coefficient of consumers, i.e., the coefficient of consumers' preference for remanufactured products, with higher values indicating that consumers are more willing to purchase remanufactured products
π_{in}^j, π_{ir}^j	Denote the profits of the original manufacturer and the remanufacturer in the two recycling channels under model j, respectively

3.3. Model functions

(1) Demand function

Referring to the research results of Chai *et al*. (2018) and Zhu *et al*. (2024), the market demand function for new and remanufactured products can be obtained as: $q_{in}^j=\frac{1-\delta-p_{in}^j+p_{ir}^j}{1-\delta}$, $q_{ir}^j=\frac{\delta p_{in}^j-p_{ir}^j}{\partial(1-\delta)}$, this leads to the derivation of the classical inverse consumer demand function for the two products as:p_{in}^j = $1-q_{in}^j-\delta q_{ir}^j$, p_{ir}^j $\delta(1-q_{in}^j-q_{ir}^j)$, where *i*∈{*OR*, *RR*, *SR*}, *j*∈{*L*, *H*}

(2) Recycling cost

There is no linear relationship between the cost of recycling and the amount of recycling, and the difficulty of recycling increases with the scale of recycling (Zhu *et al*., 2024). According to the classical recycling function in the studies of Zheng *et al*. (2021) and Huang *et al*. (2017), assuming that both new and remanufactured products can be sold and there is no inventory effect, the recycling rate of old products is $\tau_i=\frac{q_{ir}^j}{q_{in}^j}$, the ratio of the number of old products recycled to the number of new products sold. Assuming that the recycling cost is a convex function of the recycling quantity, we can get the recycling cost of used products as $\frac{k}{2}q_{ir}^{j^2}=\frac{k}{2}\tau_i^{j^2}q_{in}^{j^2}$, where k is the recycling coefficient of used products.

3.4. Model hypotheses

(1) It is assumed that manufacturers' decisions are made in a single cycle, in which the remanufacturers can obtain used products to meet the market demand for remanufacturing production. In this paper, we study the stable market, and the single-cycle model can be regarded as an infinite-period model for the study. In addition, the single-cycle model allows us better to analyze the comparative study of the recycling channels, reducing the analysis's complexity.

(2) Drawing on Chai *et al*. (2018) and Savaskan and Van Wassenhove (2006), it is assumed that the original manufacturer has sufficient channel power and is the dominant player in the supply chain, and the remanufacturer is a follower in the supply chain.

(3) Referring to the studies of Huang *et al*. (2017), it is assumed that the used products recovered by original manufacturers, remanufacturers, and retailers undergo a rigorous testing process and can all be used for remanufacturing production, and that there is a substitutability (not a complete substitution) between the new and remanufactured products in order to create competition in the market.

(4) This paper assumes a fully regulated carbon trading market, where the carbon price, as a key variable in the government's regulation of the market and the game between buyers and sellers, is guided by the government to be formed.

4. Modelling and solving

4.1. Modelling

(1) OEM Recovery Model (OR Model)

OEMs control the aftermarket channels for their products so they can recycle and reuse used products to maximize the use of resources. At the same time, OEMs have a clearer understanding of the structure and function of the product and can guarantee that the used product can be used for remanufacturing. In this model, the original manufacturer grants the right to remanufacture and sell the used product to a third-party remanufacturer by recycling the waste from the consumer. Each party determines the wholesale price of new and remanufactured products which retailers sell. For example, Apple recycles consumers' e-waste through a trade-in program and delegates the remanufacturing rights to Foxconn. Thus, the profit function for the original manufacturer, the remanufacturer, and the retailer in this model is:

Grandfathering rule (Model L):

$$\pi_{ORn}^{L}=\left(p_{ORn}^{L}-c_{n}\right)q_{ORn}^{L}+z_{OR}^{L}\cdot q_{ORr}^{L}-kq_{ORr}^{L^2}/2-\left(e_{n}q_{ORn}^{L}-E\right)p_{e} \quad (1)$$

$$\pi_{ORr}^{L}=\left(p_{ORr}^{L}-c_{r}\right)q_{ORr}^{L}-z_{OR}^{L}\cdot q_{ORr}^{L} \quad (2)$$

In equation (1) $(p_{ORn}{}^{L}-c_n)q_{ORn}{}^{L}$ represents the original manufacturer′s revenue from the production and sale of new products, $z_{OR}{}^{L}\ q_{ORr}{}^{L}$ represents the licensing fee charged by the original manufacturer to the authorised remanufacturer under the protection of intellectual property rights, $\frac{k}{2}q_{ORr}^{L^2}$ represents the recycling cost that the manufacturer needs to invest in to recycle the used products, $(e_n q_{ORn}{}^{L}-E)p_e$ represents the carbon trading situation of the original manufacturer under the grandfathering rule, when $e_n q_{ORn}{}^{L}-E>0$, it represents the need to purchase additional carbon credits from the carbon trading market in order to meet the production needs of the enterprise, when $e_n q_{ORn}{}^{L}-E<0$, it represents that the original manufacturer can sell the remaining carbon credits in the carbon trading market to benefit. In equation (2) $(p_{ORr}{}^{L}-c_r)\ q_{ORr}{}^{L}$ denotes the revenue of the remanufacturer from the production and sale of units of remanufactured products.

Table 2. Optimal solutions under the OR model

	L	H
$z_{OR}{}^{j*}$	$\frac{-c_r(-2+k)}{-4+k}+\frac{(-2+k+\frac{1}{2}(-4+k)(1-c_n-e_np_e)\delta}{-4+k}$	$\frac{-c_r(-2+k)}{-4+k}+\frac{\left(-2+k+\frac{1}{2}(-4+k)(1-c_n-(e_n-\alpha)p_e)\right)\delta}{-4+k}$
$q_{ORr}{}^{j*}$	$\frac{2c_r}{2(-4+k)\delta}+\frac{(2+c_n(-4+k)-k+e_n(-4+k)p_e)}{2(-4+k)\delta}$	$\frac{2c_r}{2(-4+k)\delta}+\frac{(2+c_n(-4+k)-k+(e_n-\alpha)(-4+k)p_e)}{2(-4+k)\delta}$
$q_{ORn}{}^{j*}$	$\frac{1-c_n-e_np_e}{2}$	$\frac{1-c_n-(e_n-\alpha)p_e}{2}$
$p_{ORr}{}^{j*}$	$\frac{-c_r+(-3+k)\delta}{-4+k}$	$\frac{-c_r+(-3+k)\delta}{-4+k}$
$p_{ORn}{}^{j*}$	$\frac{1+c_n+e_np_e-2q_r\delta}{2}$	$\frac{1+c_n+(e_n-\alpha)p_e-2q_r\delta}{2}$
$\tau_{OR}{}^{j*}$	$\frac{-2(c_r+z_{OR}^{L})-(1+c_n+e_np_e)\delta}{2(-1+c_n+e_np_e)\delta}$	$\frac{-2(c_r+z_{OR}^{H})-(1+c_n+e_np_e)\delta}{2(-1+c_n+e_np_e)\delta}$

Benchmarking rule (Model H):

$$\pi_{ORn}^{H}=\left(p_{ORn}^{H}-c_{n}\right)q_{ORn}^{H}+z_{OR}^{H}\cdot q_{ORr}^{H}-kq_{RRr}^{H^2}/2-\left(e_{n}-\alpha\right)q_{ORn}^{H}p_{e} \quad (3)$$

$$\pi_{ORr}^{H}=\left(p_{ORr}^{H}-c_{r}\right)q_{ORr}^{H}-z_{OR}^{H}\cdot q_{ORr}^{H} \quad (4)$$

(2) Remanufacturer Recovery Model (RR Model)

The remanufacturer has more specialized scrap recycling channels and comprehensive product testing technology. Under this model, the original manufacturer is responsible for the manufacture and sale of new products, and after charging a certain licensing fee, it grants to the remanufacturer the right to recycle and remanufacture the used products and the production and sale of the remanufactured products. Take BMW as an example, it has entrusted the right to recycle and remanufacture gearboxes to ZF Sales and Service (China) Co Thus under this model, the profit function for the original manufacturer, the remanufacturer and the retailer is:

Grandfathering rule (Model L):

$$\pi_{RRn}^{L}=\left(p_{RRn}^{L}-c_{n}\right)q_{RRn}^{L}+z_{RR}^{L}\cdot q_{RRr}^{L}-\left(e_{n}q_{RRn}^{L}-E\right)p_{e} \quad (5)$$

$$\pi_{RRr}^{L}=\left(p_{RRr}^{L}-c_{r}\right)q_{RRr}^{L}-z_{RR}^{L}\cdot q_{RRr}^{L}-kq_{RRr}^{L^2}/2 \quad (6)$$

Benchmarking rule (Model H):

$$\pi_{RRn}^{H}=\left(p_{RRn}^{H}-c_{n}\right)q_{RRn}^{H}+z_{RR}^{H}\cdot q_{RRr}^{H}-\left(e_{n}-\alpha\right)q_{RRn}^{H}p_{e} \quad (7)$$

$$\pi_{RRr}^{H}=\left(p_{RRr}^{H}-c_{r}\right)q_{RRr}^{H}-z_{RR}^{H}\cdot q_{RRr}^{H}-kq_{RRr}^{L^2}/2 \quad (8)$$

4.2. Model analysis

In order to obtain the optimal solution for different recycling channels under the two carbon allowance allocation methods, Lemma 1 is first given.

Lemma 1

(i) Eq. (2) is a concave function with respect to $\tau_{OR}{}^{L}$, and the optimal solution obtained through Eq. (2) is substituted into Eq. (1), which is a concave function with respect to $q_{ORn}{}^{L}$ and $z_{OR}{}^{L}$; Eq. (4) is a concave function with respect to $\tau_{OR}{}^{H}$, and the optimal solution obtained through Eq. (4) is substituted into Eq. (3), which is a concave function with respect to $q_{ORn}{}^{H}$ and $z_{OR}{}^{H}$.

(ii) Eq. (6) is a concave function with respect to $\tau_{RR}{}^{L}$, nd the optimal solution obtained through Eq. (6) is substituted into Eq. (5), which is a concave function with respect to $q_{RRn}{}^{L}$ and $z_{RR}{}^{L}$; Eq. (8) is a concave function with respect to $\tau_{RR}{}^{H}$, and the optimal solution obtained through Eq. (8) is substituted into Eq. (7), which is a concave function with respect to $q_{RRn}{}^{H}$ and $z_{RR}{}^{H}$.

The relevant proof process will not be described in detail. From the above, it is possible to draw conclusions:

Conclusion 1

The optimal solutions under the OR model are detailed in **Table 2.** The optimal solutions under the RR model are detailed in **Table 3** From **Conclusion 1, Conclusions 2 to 8** can be obtained.

The optimal solutions under the RR model are detailed in **Table 3** From **Conclusion 1**, **Conclusions 2** to 8 can be obtained.

Table 3. Optimal solutions under the RR model

	L	H
$z_{RR}{}^{j*}$	$\frac{\delta-c_r}{2}$	$\frac{\delta-c_r}{2}$
$q_{RRr}{}^{j*}$	$\frac{\delta(c_n+e_np_e)-c_r}{2(2\delta+k-\delta^2)}$	$\frac{\delta(c_n+e_np_e-\alpha p_e)-c_r}{2(2\delta+k-\delta^2)}$
$q_{RRn}{}^{j*}$	$\frac{1}{2}+\frac{\delta c_r-(2\delta+k)(c_n+e_np_e)}{2(2\delta+k-\delta^2)}$	$\frac{1}{2}+\frac{\delta c_r-(2\delta+k)(c_n+e_np_e-\alpha p_e)}{2(2\delta-\delta^2+k)}$
$p_{RRr}{}^{j*}$	$\frac{\delta}{2}+\frac{\delta(1-\delta)c_r+\delta(\delta+k)(c_n+e_np_e)}{2(2\delta+k-\delta^2)}$	$\frac{\delta}{2}+\frac{\delta(1-\delta)c_r+\delta(\delta+k)(c_n+e_np_e-\alpha p_o)}{2(2\delta+k-\delta^2)}$
$p_{RRn}{}^{j*}$	$\frac{1+c_n+e_np_e}{2}$	$\frac{1+c_n+(e_n-\alpha)p_e}{2}$
$\tau_{RR}{}^{j*}$	$\frac{\delta(c_n+e_np_e)-c_r}{(2\delta+k)(c_n-e_np_e)-\delta^2-\delta c_r}$	$\frac{\delta(c_n+c_np_e-e_np_e)-c_r}{2(\delta-k-\alpha p_e)-c_r}$

It is known that $Z_{ORn}{}^{L*} > Z_{ORr}{}^{H*}$, that is, when the original manufacturer is responsible for recycling, the cost of authorization under the grandfathering rule is higher than that under the benchmarking rule. This is because in the case where the original manufacturer is responsible for recycling, the grandfathering rule will result in insufficient allowances due to its high emission level in the past, which will in turn increase its authorization fee. In contrast, the benchmarking rule, which authorizes allowances based on the average or advanced level of the whole industry, can better reflect the effectiveness of manufacturers in reducing emissions after low-carbon transformation and alleviate the pressure on their carbon costs. This contrasts with Zhu *et al*. (2024) that the cost of remanufacturing authorizations is unaffected by government carbon trading policies, as the latter treats the cost of authorizations as a relatively rigid decision that is unresponsive to changes in carbon policy, focusing on emissions investments and production control. In contrast, we model the cost of authorization as a strategic variable, and our findings highlight how carbon policy can reshape pricing decisions across the supply chain beyond production-related costs.

Management insights: Different ways of allocating carbon allowances are not effective in changing the licensing cost decisions of OEMs. However, it is found that an increase in the demand for remanufactured products will have a positive impact on the economic performance of both original manufacturers and remanufacturers, and will also help to promote socio-economic circularity. In view of this, the Government should strengthen its support for the remanufacturing industry through measures such as lowering the taxes associated with remanufactured products, providing subsidies for purchases, and raising public awareness of the reuse of used and end-of-life products.

Conclusion 2 Comparative analysis of authorization fees for recycling channels under different allocation methods of carbon allowances:

It is known that $Z_{RRn}{}^{L*} = Z_{RRr}{}^{H*}$, different allocation of carbon allowances under the remanufacturer recycling channel does not affect the original manufacturer's remanufacturing authorization cost decisions, i.e. different allocation of carbon allowances cannot change the authorization cost of the original manufacturer, which is consistent with the conclusion of Xia *et al*. (2023). In conjunction with the relevant findings, original manufacturers tend to shift the benefits of remanufacturers by increasing the unit pricing of new products rather than adjusting the licensing fees. Selling more remanufactured products increases the profit of the original manufacturer, and increasing the unit licensing fee increases the cost of the remanufacturer, which reduces the incentive of the remanufacturer to engage in remanufacturing and leads to a reduction in the benefits for both parties.

Conclusion 3 Impact of the carbon quota allocation methodology on product prices in the two recycling channels:

(i) $p_{ORn}^{L*} > p_{ORn}^{H*}$, $p_{ORn}^{L*} = p_{ORn}^{H*}$;

(ii) $p_{RRn}^{L*} > p_{RRn}^{H*}$; $p_{RRr}^{L*} > p_{RRr}^{H*}$

Conclusion 3 shows that in the OEMs recycling channel, pricing of new products under the grandfathering rule is higher than under the benchmarking rule, while changes in the carbon allowance allocation method do not affect the price of remanufactured products. This is mainly due to additional carbon allowance purchase costs and technology adaptation costs associated with high emissions. Prices of remanufactured products, on the other hand, are not affected by the way carbon allowances are allocated. In the recycling channel of remanufacturers, prices of new and remanufactured products are higher under the grandfathering rule than under the benchmarking rule.

The main reason for this is that while the Government's carbon tax policy has forced original manufacturers to reduce carbon emissions, the grandfathering rule, which determines carbon allowances based on historical data, provides a scope of protection and fails to effectively promote emissions reductions, leading original manufacturers to increase product pricing and reduce production to meet emissions reduction targets, while at the same time increasing the pricing of remanufactured products in order to increase profits. The benchmarking rule, which determines carbon allowances based on industry carbon emission data, puts pressure on high-emission manufacturers, who tend to invest in emission reduction technologies rather than increase product pricing to cope with high carbon emission costs. As a result, both products are priced higher under the grandfathering rule.

Management insights: Under the grandfathering rule, OEMs with higher carbon emissions usually compensate for the cost of carbon emissions by increasing the unit pricing of their products and passing it on to consumers. In order to safeguard consumers' rights, the government should consider adopting the benchmarking rule, a move that not only ensures consumers' interests, but also helps to incentivize OEMs to invest in carbon-reducing technologies, thus achieving a win-win situation. In the long run, companies should also incorporate carbon reduction into their sustainable development strategies and integrate environmental protection concepts into their corporate cultures and business models in order to adapt to increasingly stringent environmental regulations and consumer preferences.

Conclusion 4 Impact of Carbon Allowance Allocation Methods on Product Sales in Two Recycling Channels:

(i) $q_{ORn}^{L} < q_{ORn}^{H}$, $q_{ORR}^{L} < q_{ORr}^{H}$;

(ii) $q_{RRn}^{L} < q_{RRn}^{H}$, $q_{RRr}^{L} > q_{RRr}^{H}$

Conclusion 4 shows that sales of new and remanufactured products under the grandfathering rule are always higher than the industry benchmarking rule when the original manufacturer is responsible for recycling. Combined with **Conclusion 2**, under the grandfathering rule, OEMs and remanufacturers will choose to raise the unit pricing of their products, while OEMs will choose to reduce the production of new products in order to reduce carbon emissions, thus weakening the purchasing behavior of consumers. When the remanufacturer is responsible for recycling, under the grandfathering rule, the original manufacturer raises the unit pricing of the new product, which leads to an increase in sales of the remanufactured product under market competition.

Management Insight: If the government adopts the grandfathering rule, it will encourage the sales of remanufactured products using the recycling channel of remanufacturers and increase the motivation of consumers to buy remanufactured products, while for remanufactured products using the recycling of the original manufacturer, the sales of remanufactured products will be reduced due to the increase in the pricing of remanufactured products per unit in the competitive market game, so the government should understand the actual situation of the enterprises and combine it with the trading mechanism of the carbon market to formulate a more targeted carbon allowance allocation method to enhance environmental benefits. Therefore, the government should understand the actual situation of the enterprises and combine the carbon market trading mechanism with the carbon market trading mechanism to formulate a more targeted carbon quota allocation method to enhance the environmental benefits.

Conclusion 5 Impact of the carbon quota allocation method on the recycling rate of used products from two recycling channels:

(i) $\tau_{OR}^{L*} > \tau_{OR}^{H*}$

(ii) $\tau_{RR}^{L*} > \tau_{RR}^{H*}$

Conclusion 5 shows that the recycling rate of used products when the Government allocates carbon allowances based on the grandfathering rule is always higher than that of the benchmarking rule. The combination of **Conclusion 3** and **Conclusion 4** shows that under the grandfathering rule, the sales volume of remanufactured products can be improved, and the incentive to recycle used products may be strengthened, thus increasing the recycling rate. In addition, under the benchmarking rule, the price of new products is lower, and thus the sales volume of new products can be improved, and under the influence of market competition, the consumer's demand for remanufactured products is reduced, so that the incentive of enterprises to recycle used products may be relatively low which leads to a decrease in the recycling rate of used and end-of-life products. However, observation of **Conclusion 4** reveals that when the manufacturer is responsible for recycling used products, the sales of remanufactured products are lower under the grandfathering rule than the benchmarking rule, but the recycling rate of used products is higher, which is because the original manufacturer under the grandfathering rule may participate in the recycling of used products more actively in order to obtain more carbon allowances, as this can assist in proving that it is taking measures to reduce carbon emissions. Therefore, although sales of remanufactured products are lower, the recycling rate of used products may be higher due to the increased incentive to recycle used products.

Management insights: When the original manufacturer is responsible for recycling used products, in order to avoid a situation where the original manufacturer passively recycles more used products in order to obtain more carbon quotas and a backlog of remanufactured products occurs, the Government should take into account the enterprises' responses when setting carbon quotas and adopt a more appropriate carbon quota allocation method. When remanufacturers are responsible for recycling used products, in order to encourage the development of the remanufacturing industry, the Government should set a lower carbon quota to limit the production activities of the original manufacturers so as to promote the sale of remanufactured products.

Governments should adopt carbon tax policies to limit the negative environmental impacts of manufacturing processes. Therefore, in order to analyze the environmental impacts of the different allocation of carbon allowances in the two recycling channels, the environmental impacts of new and remanufactured products are considered to be: $e_i^j = e_n q_{in}^j + e_r q_{ir}^j$

Conclusion 6 Environmental impacts of two recycling channels under different allocation of carbon allowances:

(i) $e_{OR}^L < e_{OR}^H$;

(ii) $e_{RR}^L < e_{RR}^H$

Comparison with Savaskan and Van Wassenhove (2006)' s research, this paper further talks about the environmental impacts of different ways of carbon quota allocation under different recycling channels, taking corporate responsibility and government benefits into view. **Conclusion 6** shows that the environmental impacts of both recycling channels under the grandfathering rule are always weaker than those of the benchmarking rule. This is because the volume of product sales is directly linked to the environmental impacts, and **Conclusion 4** shows that the volume of product sales under the grandfathering rule is always lower than that of the benchmarking rule. However, we find a situation in (ii) of **Conclusion 4** and (ii) of **Conclusion 6**: i.e., in the recycling channel of remanufacturers, the sales volume of remanufactured products under the grandfathering rule is lower than that of the benchmarking rule, but the impact on the environment is lower, which is due to the fact that the difference in the sales volume of new products under the different allocation methods is greater than that of remanufactured products, which means that the impact on the environment caused by the new products is greater than that caused by the remanufactured products. This is because the difference in sales of new products is greater than the difference in sales of remanufactured products under the different allocation methods. Combined with the sales volume, we can see that the environmental impact of the grandfathering rule is lower than that of the benchmarking rule.

Management insights: Strict allocation of carbon allowances under a carbon trading policy can have a significant impact on both the economic and environmental benefits of enterprises, which means that the government should develop a mechanism for allocating carbon allowances that achieves a "win-win" situation. The baseline law provides an incentive for companies to take more proactive environmental measures to reduce their carbon emissions in order to comply with the baseline requirements and obtain additional carbon allowances. This competitive mechanism helps to promote environmental innovation and technological advancement, improve resource utilization efficiency and reduce negative impacts on the environment. It can also increase the sales volume of recycled products and consumer demand for recycled products, thereby promoting the development of the recycling industry.

Conclusion 7 Impact of Carbon Allowance Allocation Methods on Manufacturers' and Remanufacturers' Profits:

(i) $\frac{\partial \pi_{ORn}^{L^*}}{\partial E} > 0$, $\frac{\partial \pi_{ORr}^{L^*}}{\partial E} = 0$, $\frac{\partial \pi_{ORn}^{H^*}}{\partial \alpha} > 0$, $\frac{\partial \pi_{ORr}^{H^*}}{\partial \alpha} < 0$

(ii) $\frac{\partial \pi_{RRn}^{L^*}}{\partial E} > 0$, $\frac{\partial \pi_{RRr}^{L^*}}{\partial E} = 0$, $\frac{\partial \pi_{RRn}^{H^*}}{\partial \alpha} > 0$, $\frac{\partial \pi_{RRr}^{H^*}}{\partial \alpha} < 0$

(iii) $\pi_{ORn}^{L*} > \pi_{ORn}^{H*}$ when

$$e_n > \frac{4c_r + \left(4 + 2c_n(-4+k) + 4p_e\alpha - k(2+p_e\alpha)\right)\delta}{2(-4+k)p_e\delta}$$

(iv) $\pi_{ORr}^{L*} > \pi_{ORr}^{H*}$ when

$$e_n > \frac{4c_r k(-1+\delta) + \delta(k^2(2+p_e\alpha) - 2c_n(-4+k)(-2+k-4\delta)}{2(-4+k)p_e(-2+k-4\delta)\delta} + \frac{8(2+p_e\alpha)(1+2\delta) - 2k\left(4+6\delta+p_e\alpha(3+2\delta)\right)}{2(-4+k)p_e(-2+k-4\delta)\delta}$$

(v) $\pi_{RRn}^{L*} > \pi_{RRn}^{H*}$ when

$$\frac{E}{\alpha} > \frac{1}{4}\left(2 + \frac{2c_r\delta - 2(c_n + e_n p_e)(k+2\delta) + p_e\alpha(k+2\delta)}{k - (-2+\delta)\delta}\right)$$

(vi) $\pi_{RRr}^{L*} > \pi_{RRr}^{H*}$

Conclusion 7 shows that: in the original manufacturer recycling channel, the profit of the original manufacturer increases with the total amount of carbon allowances, while the profit of the remanufacturer is not related to the total amount of carbon allowances under the grandfathering rule and decreases with the increase of carbon allowances under the benchmarking rule; in the recycling channel of the remanufacturer, the profit of the remanufacturer is not directly related to the total amount of carbon allowances under the grandfathering rule and is a decreasing function of total amount of carbon allowances under the benchmarking rule, while the profit of the original manufacturer always has an increasing function. In the remanufacturer recycling channel, the profit of the remanufacturer is not directly related to the total amount of carbon allowances under the grandfathering rule and is a decreasing function of the total amount of carbon allowances under the benchmarking rule, while the profit of the original manufacturer is always an increasing function. It is worth noting that the profit of remanufacturers under the grandfathering rule is higher than that of the benchmarking rule. We find that the profit of the original manufacturer always increases with the increase of the total amount of carbon allowances, and the combination of **Conclusion 3** and **Conclusion 4** shows that the larger the total amount of carbon allowances, the slightly lower the government's carbon emission requirement for the enterprise, and the more new products the enterprise produces and sells. In addition, we find that for the comparison of profits under the two methods, there is a relationship with the unit carbon emissions of new products.

Management Insight: With the increasing environmental pollution problem, the government has become increasingly strict in regulating carbon emissions of enterprises. This means that companies will face a gradually decreasing total initial carbon allowance allocation, which will directly affect their profitability. To cope with this challenge, companies need to increase their investment in carbon reduction areas to lower the carbon emissions of their products and maintain a competitive edge in the carbon emissions market. For manufacturers, it's time to take action to reduce carbon emissions. At the same time, co-operating with fellow companies and investing in carbon reduction projects will help to reduce costs, improve efficiency and move the industry towards more sustainable development. Such co-operation will not only help to meet the government's stringent requirements on carbon emissions, but will also bring long-term economic and environmental benefits to businesses.

5. Numerical analysis

5.1. Parameter settings and data sources

In order to further validate the conclusions of this study, and to deeply analyze the green consumption preference of consumers who are affected by the government's promotion of green consumption concepts, combined with the changes in the scale of recycling by enterprises due to the change in consumers' concepts, we study the changes in the decision-making behaviors of each supply chain participant in terms of the pricing sales of the remanufactured products, as well as the changes in the licensing decision-making costs of the original manufacturer.

This paper takes Volkswagen FAW (Dalian) as the empirical object, considering the investment problem of remanufacturing technology, and authorizes the recycling and remanufacturing production of EA888 engine to Volkswagen FAW (Dalian) Co. Ltd, which means that the remanufacturer is in charge of recycling the used engine and producing and selling it. In this paper, MATLAB is used for numerical simulation and analysis. According to the data released by China Association of Automobile Manufacturers (CAAM) on remanufacturing of cycling parts, remanufactured engines can save 50% of the production cost compared with the new engines, so we assume that $c_n = 0.2$, $c_r = 0.1$.

Other model parameters are set with reference of Xia *et al*. (2023) and Zhu *et al*. (2024). and typical values from domestic carbon tax policies. Specifically, we set the carbon price $p_e = 0.3$ the baseline carbon allowances $\alpha = 0.4$, the unit carbon emission of new products $e_n = 1$ recovery scale factor $k \in (0, 1]$ consumer green preference factor $\delta \in (0, 1]$. While no raw firm-level data is used, the parameter settings follow standard theoretical modeling practices to ensure comparability and consistency with existing studies.

5.2. Impact of δ and k on unit mandate costs

As can be seen from the left panel of **Figure 2**, in the OEM recycling channel, the grandfathering rule mitigates to a certain extent the cost fluctuations due to the expansion of recycling scale and the enhancement of consumers' green preference. This is because the grandfathering rule allows firms to enjoy a more generous allocation of allowances at the initial stage, but the further expansion of the recycling scale with increasing green preference leads to complex non-linear changes in the authorization costs.

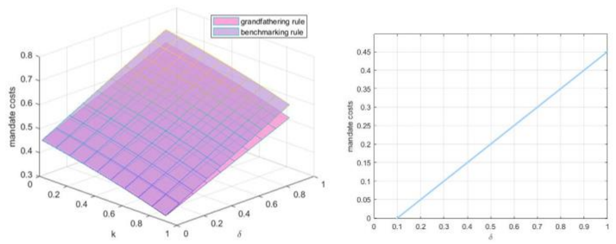


Figure 2. Impact of δ and k on the cost of unit authority under the historical and baseline approaches

As can be seen from the right panel of **Figure 2**, in the remanufacturer recycling channel, the unit authorization cost is the same under both carbon allowance allocation methods. However, the unit authorization cost is positively correlated with consumers' green consumption preference, which is because the increase in demand for remanufactured products will cause a certain impact on the original manufacturer, and the profit of the original manufacturer will be reduced by it, and in order to increase the profit of the enterprise, the original manufacturer tends to increase the production cost of the remanufacturer through the increase of the unit authorization cost, which makes the remanufacturer increase the unit retail price of the product in order to reduce its Growing market demand

5.3. Impact of δ and k on unit retail prices of remanufactured products

As can be seen from **Figure 3**, there is no significant difference between the two carbon quota allocation methods in the recycling channel of original manufacturers, which suggests that when original manufacturers have strong control and integration advantages in the recycling process, they can smoothly meet the carbon quota requirements through internal synergy and overall control. The retail price of remanufactured products is positively correlated with the recycling scale factor and consumers' green consumption preference under the two allocation methods of carbon allowances, because when the demand for remanufactured products increases, the remanufacturer needs more resources and labor to recycle, upgrade and produce the remanufactured products, and these additional costs lead to the rise in the cost of remanufactured products. In addition, as consumers' green consumption preference increases, the licensing fee also increases gradually, so it can be seen that the original manufacturer will control the unit licensing fee decision in order to put more cost pressure on the remanufacturer, and in order to safeguard the profit, the remanufacturer will have to respond to this by raising the unit retail price of the remanufactured product in order to safeguard the profit in the face of the increasing cost pressure and competition in the marketplace.

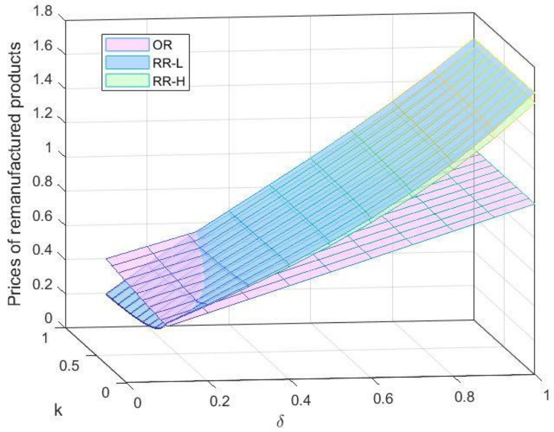


Figure 3. Impact of δ and k on unit retail prices of remanufactured products under the historical and baseline approaches

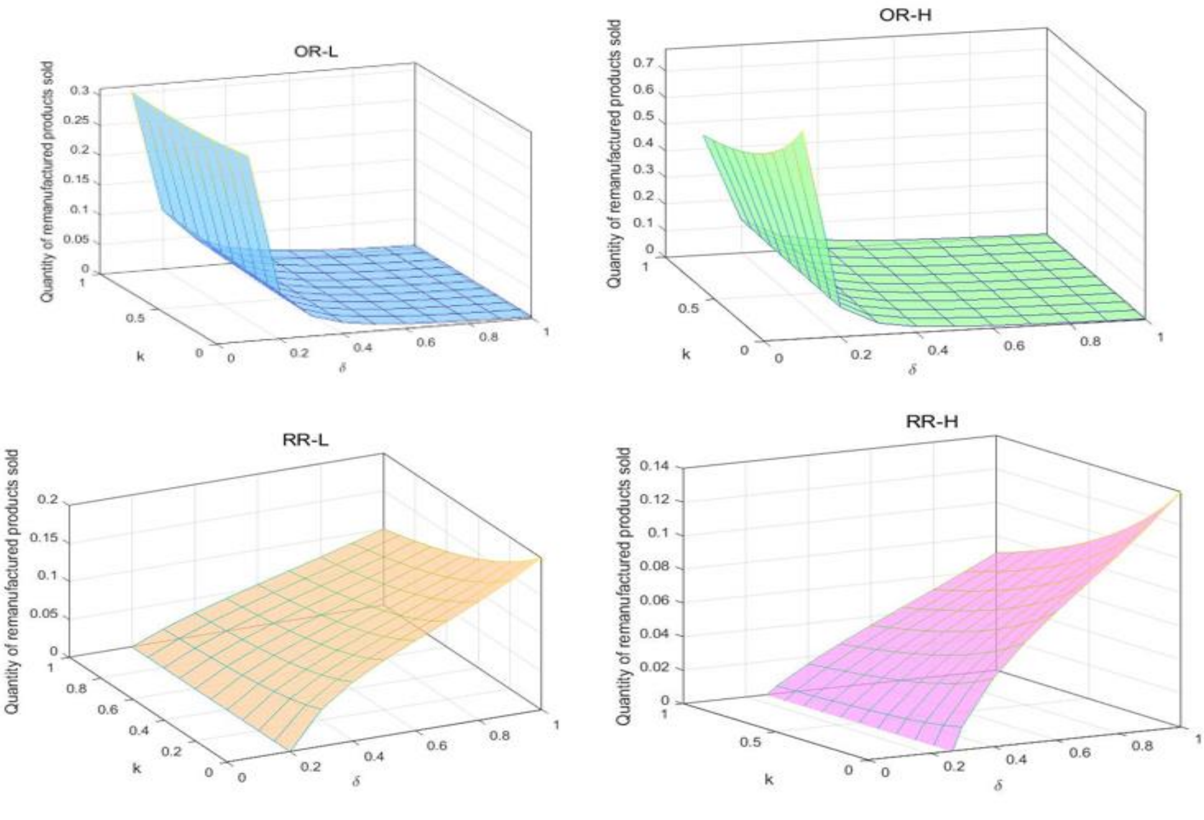


Figure 4. Impact of δ and k on the quantity of remanufactured products sold under the historical and baseline methods

5.4. Impact of δ and k on the quantity of remanufactured products sold

As can be seen in **Figure 4**, the sales volume of remanufactured products in the original manufacturer's recycling channel, both in the grandfathering rule (OR-L) and the industry benchmarking rule (OR-H), tends to decrease with the increase of the recycling scale factor k and the enhancement of the consumer's green consumption preference δ, which suggests that, in the case of the original manufacturer's control of the recycling channel, the larger scale of recycling and the higher green preference may lead to the remanufactured products to be less competitive in the market. In addition, the downward trend is more obvious in the historical emissions rule compared to the industry benchmarking rule, which suggests that under the historical emissions rule, the original manufacturer faces greater pressure on carbon costs under high recycling scale, and transfers production costs to the remanufacturer through the licensing fee, which inhibits the sales of remanufactured products.

Sales of remanufactured products in the remanufacturer's recycling channel increase significantly with an increase in the recycling scale factor k and an increase in consumers' green consumption preference δ. However, the industry benchmarking rule shows a smaller increase in sales relative to the historical grandfathering rule, which may be due to the fact that the historical grandfathering rule provides more flexibility in the recycling channel when the remanufacturer controls the recycling channel, allowing the remanufacturer to utilize the recycling resources more efficiently, which leads to an increase in sales of remanufactured products.

The numerical analysis results show that regardless of the carbon quota allocation mode adopted by the government, the increase of consumer green preference δ and recycling scale factor k will have a significant positive effect on the price of remanufactured products; meanwhile, under the environment of market competition game, original manufacturers will increase the licensing fee accordingly in order to transfer the cost pressure brought by the carbon quota policy. This phenomenon shows obvious differences under different recycling channel modes: when the original manufacturer controls the recycling channel, the sales of remanufactured products, on the contrary, show a decreasing trend with the increase of consumer green preference and recycling scale factor. This implies that in the case of higher recycling cost or higher consumer green preference, the cost pass-through effect triggered by the increase of licensing fee by the OEM will significantly inhibit the competitiveness of the remanufacturer in the market. Therefore, under the market environment of high cost and strong green preference, the OEM-led recycling channel selection needs to carefully consider the strategic setting of licensing fees to avoid the sales decline of remanufactured products being too obvious. And when the recycling channel is dominated by the remanufacturer, the sales volume of remanufactured products increases with δ and k increase significantly, suggesting that under this channel model, remanufacturers can more effectively cope with high-cost or strong green preference environments and realize market share expansion. In summary, the numerical simulations in this study highlight the significant effects of changes in consumer preferences and recycling costs on the strategic choices of different recycling channels, and the model conclusions are most robust in the context of moderate consumer preferences and moderate recycling costs; under extreme parameter condition, firms need to be more cautious in applying the strategic recommendations proposed in this paper.

6. Conclusions, limitation and research prospects

6.1. Conclusions

This paper constructs a closed-loop supply chain consisting of an original manufacturer and a remanufacturer under an authorized remanufacturing model, based on the original manufacturer recycling channel and the remanufacturer recycling channel respectively, and compares the impacts of unit authorization cost, product price, sales volume, recycling rate of used products, the environment and the profit of the enterprise according to the two ways of allocating the government's carbon allowances: grandfathering rule and the benchmarking rule, so as to provide enterprises with the optimal choice of recycling channels from the perspectives of both environmental and economic benefits, to provide enterprises with the optimal choice of recycling channels. The main conclusions of this paper are as follows:

(1) The grandfathering rule set by the Government is not effective in promoting original manufacturers to invest in carbon-reducing technologies. Instead, OEMs tend to meet emission reduction requirements by raising product prices or reducing production, rather than proactively adopting cleaner technologies. Moreover, under competitive market pressure, they may also increase the pricing of new products to enhance profitability. In contrast, the benchmarking method allocates carbon allowances based on industry-wide emission standards, imposing greater pressure on high-emission enterprises. Under the combined constraints of carbon trading and market competition, these firms are more inclined to invest in low-carbon technologies rather than simply shifting carbon costs through price increases.

(2) Whether or not the way carbon allowances are allocated affects the decision to authorize fees depends on the dominant party in the recycling channel. When recycling is the responsibility of the remanufacturer, the authorization fee set by the original manufacturer remains the same under different carbon allowance policies, in which case the original manufacturer prefers to adjust the pricing of the new product to make profit rather than playing the game by modifying the authorization fee. However, when the original manufacturer assumes responsibility for recycling, the situation changes significantly: the authorization fee under the grandfathering rule is higher than that under the baseline rule. This is due to the fact that the grandfathering rule allocates allowances based on past emission records, which results in higher carbon constraints on OEMs, which tend to increase the authorization fee to pass on the cost, while the benchmarking rule allocates allowances based on a uniform standard for the benchmarking rule, which is a better reflection of their emission reduction effectiveness and eases the burden of carbon costs.

(3) The carbon allocation mechanism significantly affects the preference for recycling channels. This preference is largely due to the OEMs’ strategic responses under this allocation scheme, where they are more likely to raise

unit product prices and reduce output to meet carbon reduction targets. The increase in new product prices enhances the market competitiveness of remanufactured products, thereby boosting their sales and encouraging remanufacturers to intensify efforts in collecting end-of-life products. Within this carbon quota framework, remanufacturer-led recycling not only supports corporate carbon mitigation but also facilitates resource reuse and improves product circularity. As a result, this approach yields both environmental and economic benefits. Therefore, the historical emission method plays a positive role in guiding enterprises toward remanufacturer-based recycling strategies and contributes meaningfully to the advancement of a circular economy.

(4) Numerical analysis shows that, regardless of the carbon allowance allocation method adopted by the government, an increase in consumer green preference (δ) and recycling scale factor (k) positively affects the price of remanufactured products and simultaneously drives OEMs to increase licensing fees. In the remanufacturer-led recycling scenario, our results are consistent with Zhu *et al*. (2024), who conclude that licensing fees remain unaffected by the carbon allocation mechanism. However, when considering the OEM-led recycling scenario, our results diverge significantly from Zhu *et al*.'s findings. We observe that the licensing fee varies considerably with the carbon allowance allocation method, with higher fees under the benchmarking rule compared to the grandfathering rule. This is due to the stronger carbon cost pressure imposed by benchmarking, prompting OEMs to transfer costs via licensing fees. Furthermore, OEM-led recycling offers greater strategic flexibility under dual pressures from carbon regulation and market competition, enabling OEMs to better manage remanufacturing competition and optimize supply chain profits.

6.2. Limitations and research prospects

Although this study has made some progress based on existing research, there are still some limitations. Although this paper has made initial progress in theoretical modeling and numerical simulation, there are still some limitations. First, the modeling in this paper is based on a single-cycle static Stackelberg game model, which does not take into account the evolution of recycling behavior in a multi-cycle dynamic situation, and may underestimate the room for long-term strategy adjustment. Second, key parameters such as consumers' green preference and recycling scale are mainly set with reference to the existing literature. Although this is a common modeling approach in theoretical research, which ensures the consistency of the model in terms of resolvability and comparison with the literature, it also lacks the empirical calibration of actual enterprise data, and the extrapolation of conclusions still needs to be further strengthened. In addition, this paper assumes complete information symmetry and does not introduce game complexity factors such as information asymmetry and behavioral preferences, which are more common in reality.

Future research can be carried out in the following aspects: first, constructing a multi-cycle dynamic decision model to analyze the dynamic evolution of recycling channels and pricing strategies; second, introducing uncertainty factors, such as carbon price fluctuations, recycling cost perturbations, etc., to enhance the adaptability and robustness of the model; third, combining with the actual enterprise or industry data, empirically verifying the model to enhance the realistic feasibility and policy guidance value of the conclusions; Fourth, we will further explore the collaborative recycling strategy after the intervention of platform-type enterprises or third-party data platforms, so as to expand the research space of the closed-loop supply chain in the digital economy environment.

References

Chai, Q., Xiao, Z., Lai, K.-h., and Zhou, G. (2018). Can carbon cap and trade mechanism be beneficial for remanufacturing? *International Journal of Production Economics*, **203**, 311–321

Ghosh, S. K., Seikh, M. R., and Chakrabortty, M. (2020). Analyzing a stochastic dual-channel supply chain under consumers' low carbon preferences and cap-and-trade regulation. *Computers & Industrial Engineering*, **149**.

Gu, K., Tokoro, C., Takaya, Y., Zhou, J., Qin, W., and Han, J. (2024). Resource recovery and regeneration strategies for spent lithium-ion batteries: Toward sustainable high-value cathode materials. *Waste Management*, **179**, 120–129.

Hong, Z. F., Chu, J., Zhang, L. L., and Wang, N. N. (2024). Recycling Channel Selection for a Manufacturer Involving Consumers' Green-Return Behavior. *Ieee Transactions on Engineering Management*, **71**, 8761–8776.

Hu, X., Yang, Z., Sun, J., and Zhang, Y. (2020). Carbon tax or cap-and-trade: Which is more viable for Chinese remanufacturing industry? *Journal of Cleaner Production*, **243**, 118606.

Huang, G. X., Ding, Q., Dong, C. W., and Pan, Z. C. (2021). Joint optimization of pricing and inventory control for dual-channel problem under stochastic demand. *Annals of Operations Research*, **298**(1–2), 307–337.

Huang, M., Yi, P., and Shi, T. (2017). Triple Recycling Channel Strategies for Remanufacturing of Construction Machinery in a Retailer-Dominated Closed-Loop Supply Chain. *Sustainability*, **9**(12), 2167.

Ji, J. N., Zhang, Z. Y., and Yang, L. (2017). Comparisons of initial carbon allowance allocation rules in an O2O retail supply chain with the cap-and-trade regulation. *International Journal of Production Economics*, **187**, 68–84.

Jiao, J., Pan, Z., and Li, J. (2023). Effect of carbon trading scheme and technological advancement on the decision-making of power battery closed-loop supply chain. *Environmental Science and Pollution Research*, **30**(6), 14770–14791.

Kadeer, A., Yang, J. H., and Zhao, S. Y. (2024). Complexity Analysis of the Interaction between Government Carbon Quota Mechanism and Manufacturers' Emission Reduction Strategies under Carbon Cap-and-Trade Mechanism. *Sustainability*, **16**(16).

Kushwaha, S., Chan, F. T. S., Chakraborty, K., and Pratap, S. (2022). Collection and remanufacturing channels selection under a product take-back regulation with remanufacturing target. *International Journal of Production Research*, **60**(24), 7384–7410.

Li, B., Xia, X., and LI, Q. Research on the influence of carbon allowance trading on the improvement of carbon emission reduction technology and the coordination mechanism. *Chinese Journal of Management Science*, 1-16.

Li, X. X., Meng, M., Hong, Y. G., and Chen, J. (2024). A survey of decision making in adversarial games. *Science China-Information Sciences*, **67**(4).

Liu, B. L., Ding, C. J., Ahmed, A. D., Huang, Y. J., and Su, Y. Q. (2024). Carbon emission allowances and green development efficiency. *Journal of Cleaner Production*, **463**.

Lu, R., & Li, N. (2016). Take-back channel selection of closed-loop supply chain for anelectronic product. *Systems Engineering – Theory & Practice*, **36**(7), 1687–1695.

Quan, W., Yan, K., Zhang, Z., Nie, H., Wang, R., & Xu, Z. (2024, 2024-10-01). Novel targeted extraction of lithium: An environment-friendly controlled sulfidation roasting technology and mechanism for recovering spent lithium-ion batteries. *Separation and Purification Technology*, **345**, 127415.

Savaskan, R. C., Bhattacharya, S., and Van Wassenhove, L. N. (2004). Closed-Loop Supply Chain Models with Product Remanufacturing. *Management Science*, 50(2), 239–252.

Savaskan, R. C., and Van Wassenhove, L. N. (2006). Reverse Channel Design: The Case of Competing Retailers. *Management Science*, **52**(1), 1–14.

Savaskan, R. C., and Van Wassenhove, L. N. (2006). Reverse Channel Design: The Case of Competing Retailers. *Management Science*, **52**(1), 1–14.

Shu, T., Wu, Q., Chen, S., Wang, S., Lai, K. K., and Yang, H. (2017). Manufacturers'/remanufacturers' inventory control strategies with cap-and-trade regulation. *Journal of Cleaner Production*, **159**, 11–25.

Wang, W. B., Zhou, C. Y., and Li, X. Y. (2019). Carbon reduction in a supply chain via dynamic carbon emission quotas. *Journal of Cleaner Production*, **240**.

Wu, W. Q., Li, M., and Huang, G. Q. (2025). Optimal Recovery Mode for New Energy Vehicle Battery Recycling Under Government Policies. *Managerial and Decision Economics*.

Xia, X., Chen, j., Zhu, Q., and Li, B. (2023). Studying on the impact of carbon allowance allocation methods on authorization remanufacturing and coordination mechanism. *Systems Engineering–Theory & Practice*, **43**(9), 2632–2655.

Xia, X., Wang, Z., and Wang, W. (2024). An Evolutionary Game Analysis of Governments' Carbon Allowance Allocation and the Selection of Remanufacturing Modes Under Intellectual Property Protection. *Journal of Systems & Management*.

Xia, X. Q., Li, J. W., Wei, W., Benkraiem, R., and Abedin, M. Z. (2025). Emission reduction levels of manufacturers under carbon trading policies. *Energy Economics*, **141**.

Yang, Y., Lin, J., Hedenstierna, C. P. T., and Zhou, L. (2023). The more the better? The impact of the number and location of product recovery options on the system dynamics in a closed-loop supply chain. *Transportation Research Part E: Logistics and Transportation Review*, **175**, 103150.

Yu, Z., Qinghua. (2024). Comparative Analysis of Government Carbon Tax Policy on Three Remanufacturing Modes. *Operations research and management science* **4**, 56–66.

Zheng, B. R., Chu, J., and Jin, L. (2021). Recycling channel selection and coordination in dual sales channel closed-loop supply chains. *Applied Mathematical Modelling*, **95**, 484–502.

Zhou, Q., Meng, C., and Yuen, K. F. (2020). The impact of secondary market competition on refurbishing authorization strategies. *International Journal of Production Economics*, **228**, 107728.

Zhou, S. J., and Shan, F. X. (2023). Discovery of innovation effect and spillover effect: Evidence from intelligent manufacturing promoting low-carbon development. *Journal of Innovation & Knowledge*, **8**(3).

Zhu, Q., Xia, X., Li, M., and Wu, R. (2024). The effect of carbon allowance allocation methods on authorized remanufacturing. *Journal of Management Sciences in China*, **27**(5), 60–75.

Appendix

Proof of lemma 1

Substituting $p_{ORn}{}^L = 1 - q_{ORn}{}^L - \delta q_{ORr}{}^L$, $p_{ORr}{}^L = \delta(1 - q_{ORn}{}^L - q_{ORr}{}^L)$, $q_{ORr}{}^L = \tau_{ORn}{}^L q_{ORn}{}^L$ into Eq. (2), one obtains:

$$\pi_{ORr}^L = \left[\delta\left(1 - q_{ORn}^L - q_{ORr}^L\right) - c_r\right]\tau_{ORn}^L q_{ORn}^L - z_{OR}^L \tau_{OR}^L q_{ORn}^L \qquad (A.1)$$

Taking first-order and second-order partial derivatives of Eq. (A.1) with respect to $\tau_{OR}{}^L$ respectively, yields:

$$\frac{\partial \pi_{ORs}^L}{\partial \tau_{OR}^L} = -q_{ORn}^L\left(c_r + z_{OR}^L + \delta\left(-1 + q_{ORn}^L + 2\tau_{OR}^L q_{ORn}^L\right)\right)$$

$$\frac{\partial^2 \pi_{ORs}^L}{\partial (\tau_{OR}^L)^2} = -2\delta q_{ORn}^{L^2} < 0$$

Since its second order derivative is 0, equation (2) is concave with respect to $\tau_{OR}{}^L$.

Letting the first-order partial derivative be 0, we get $\tau_{OR}^L = \frac{c_r + z_{OR}^L + \delta\left(-1 + q_{ORn}^L\right)}{2q_{ORn}^L}$, nd substituting this into Eq. (1), we get:

$$\pi_{ORn}^L = \left(1 - q_{ORn}^L - \delta\frac{c_r + z_{OR}^L + \delta\left(-1 + q_{ORn}^L\right)}{2} - c_n\right)q_{ORn}^L$$
$$+ z_{OR}^L \frac{c_r + z_{OR}^L + \delta\left(-1 + q_{ORn}^L\right)}{2} \qquad (A.2)$$
$$- \frac{k}{2}\frac{(c_r + z_{OR}^L + \delta\left(-1 + q_{ORn}^L\right))^2}{4} - \left(e_n q_{ORn}^L - E\right)p_e$$

The first and second order partial derivatives of Eq. (A.2) with respect to $z_{OR}{}^L$ and $q_{ORn}{}^L$ are obtained:

$$\frac{\partial \pi_{ORn}^L}{\partial z_{OR}^L} = \frac{-c_r\left(-2 + k\right) + \left(-2 + k + \left(-4 + k\right) - q_{ORn}^L\right)\delta}{-4 + k}$$

$$\frac{\partial \pi_{ORn}^L}{\partial q_{ORn}^L} = q_{ORn}^L - \frac{4\left(-1 + c_n + e_n p_e + 2\right)q_{ORn}^L}{8 + \left(-4 + k\right)\delta^2}$$

$$\frac{\partial^2 \pi_{ORn}^L}{\partial (q_{ORn}^L)^2} = -2 - \frac{1}{4}\left(-4 + k\right)\delta^2, \frac{\partial \pi_{ORn}^L}{\partial z_{OR}^L q_{ORn}^L} = \frac{1}{4}\left(-4 + k\right)\delta$$

$$\frac{\partial \pi_{ORn}^L}{\partial q_{ORn}^L z_{OR}^L} = \frac{1}{4}\left(-4 + k\right)\delta \frac{\partial^2 \pi_{ORn}^L}{\partial (z_{OR}^L)^2} = 1 - \frac{k}{4}$$

The Hessian matrices of Eq. (A.2) with respect to $z_{OR}{}^L$ and $q_{ORn}{}^L$ are:

$$H=\begin{pmatrix} -2-\frac{1}{4}(-4+k)\delta^2 & \frac{1}{4}(-4+k)\delta \\ \frac{1}{4}(-4+k)\delta & 1-\frac{k}{4} \end{pmatrix}$$

$$|H|=\frac{1}{2}(-4+k)<0$$

The main diagonal elements are all less than 0, so equation (1) is a concave function with respect z_{OR}^{L} and q_{ORn}^{L}.

Similarly we can prove **Lemma (ii)** by reverse induction

Proof of conclusion 1

Due to $\frac{\partial \pi_{ORn}^{L}}{\partial z_{OR}^{L}}=\frac{-c_r(-2+k)+\left(-2+k+(-4+k)-q_{ORn}^{L}\right)\delta}{-4+k}$

and

$$\frac{\partial \pi_{ORn}^{L}}{\partial q_{ORn}^{L}}=\frac{4-4c_n-4e_n p_e+\delta\left(c_r(-2+k)+(-4+k)z_{OR}^{L}-(-2+k)\delta\right)}{8+(-4+k)\delta^2}$$

so,

$$z_{OR}^{L^*}=\frac{-c_r(-2+k)+\left(-2+k+\frac{1}{2}(-4+k)(1-c_n-e_n p_e)\right)\delta}{-4+k},$$

$$q_{ORn}^{L^*}=\frac{1-c_n-e_n p_e}{2}$$

Substituting this into the expression $\tau_{OR}^{L}=\frac{c_r+z_{OR}^{L}+\delta\left(-1+q_{ORn}^{L}\right)}{2q_{ORn}^{L}}$ gives:

$$\tau_{OR}^{L^*}=\frac{-2c_r+\left(-2-c_n(-4+k)+k-e_n(-4+k)p_e\right)\delta}{(-4+k)(-1+c_n+e_n p_e)\delta}$$

Solving in this loop we can get q_{ORn}^{L*}, q_{ORr}^{L*}, p_{ORn}^{L*}, p_{ORr}^{L*}, and substituting them into Eq. (1) and Eq. (2) we can get the optimal profit values of π_{ORn}^{L*} and π_{ORr}^{L*} for the original manufacturer and the remanufacturer. The above is the proof process of **Conclusion 1**. The rest of the model proof process is similar and will not be repeated.

Proof of conclusion 2

$$(1) z_{OR}^{L^*}=\frac{-c_r(-2+k)+\left(-2+k+\frac{1}{2}(-4+k)(1-c_n-e_n p_e)\right)\delta}{-4+k}$$

So, one gets: $z_{OR}^{L*}-z_{RR}^{L*}=0$, $z_i^{H*}=z_i^{L*}$

The proof process for **Conclusions 3 to 8** is like that of **Conclusion 1** and will not be repeated in detail.