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WATER MASS BALANCE IN THE CASE OF VERTICAL INFILTRATION

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ABSTRACT

Water movement in the unsaturated zone is an important hydrological process. Richard's equation is windily used to describe both soil water infiltration and soil water absorption. Various methods have been developed to solve Richard's equation. Wang *et al*. (2003) have developed an algebraic model for the description of soil water infiltration, based on Parlange's solution of Richard's equation and on soil retention curve and hydraulic conductivity equation given by Brooks and Corey. Their model utilizes experimental measurements of the cumulative infiltration volume and the wetting front distance as functions of time in order to describe soil water infiltration. The objective of this paper is to test the accuracy of the Wang *et al*. algebraic model for the one-dimensional (vertical) soil water infiltration. A vertical infiltration experiment was conducted on a sandy soil, for the measurement of the cumulative infiltration volume and the wetting front distance. Soil water content was determined at selected times and positions, using gamma ray absorption. Additionally the hydraulic conductivity $K(\theta)$ and the soil retention curve $\Psi(\theta)$ were determined. The algebraic model developed by Wang *et al*., was found simple to use since the required data are the cumulative infiltration (F), the wetting front distance (z_f) and the initial and saturated soil water content (θ_i and θ_s respectively). The results show a fair agreement between calculated and measured values on soil water content profiles, hydraulic conductivity and on the water mass balance.

KEYWORDS: water mass balance, vertical infiltration, hydraulic conductivity, cumulative infiltration, cumulative water volume, water content distribution

1. INTRODUCTION

Soil water infiltration is a hydrological process of high importance. The vertical transient soil water infiltration in unsaturated soils is described by the widely used Richard's equation. A series of methods has been developed in order to describe soil water infiltration or estimate soil water content distributions, namely: a) semi-analytical (Parlange, 1971; Parlange *et al*., 1999 etc), b) finite difference (Ashcroft *et al*., 1962; Celia *et al*., 1990 etc), c) finite elements (Tzimopoulos, 1978; Antonopoulos, 2000 etc), d) flux-concentration (Philip, 1973; Evangelides *et al*., 2005 etc) and e) finite control volumes (Arampatzis, 2000). Although the solutions obtained with these methods help on the description of soil water infiltration and soil water content distribution, it is very difficult to determine the required parameters. On the other hand all methods don't have the ability to make estimations. Wang *et al*. (2002) formulated a prediction method for the parameters of Brooks and Corey (1964) model using horizontal infiltration data, while the same authors (Wang *et al.*, 2003) established a simple algebraic model for the description and estimation of the one-dimensional (vertical) transient soil water infiltration. This model is based on Parlange's solution of Richard's equation, and requires simple experimental measurements (cumulative infiltration, wetting front distance as functions of time and the initial and the saturated soil water content).

The objective of this paper is to test the accuracy of the Wang *et al*. (2003) algebraic model for the one-dimensional (vertical) transient soil water infiltration. The experiments were carried out in the laboratory of Agricultural Hydraulics (School of Polytechnic – Department of Rural and Surveying Engineering – Aristotle University of Thessaloniki). Cumulative infiltration was measured by weighting the water entering the soil column, while wetting front distance was measured temporally in specific positions, using γ- ray absorption.

2. THEORY

According to Brooks and Corey (1964), the soil water retention curve can be described by the following equation:

$$\frac{\theta-\theta_r}{\theta_s-\theta_r}=\left(\frac{h_d}{h}\right)^N \tag{1}$$

where θ is the soil water content (cm^3 cm^{-3}), θ_s and θ_r are the saturated and the residual water content respectively (cm^3 cm^{-3}), h_d is the air entry pressure (cm), h is the soil water suction (cm), N is the pore size distribution coefficient. According to the above authors, the unsaturated hydraulic conductivity can be expressed as follows:

$$K(h)=K_s\left(\frac{h_d}{h}\right)^N \tag{2}$$

$$K(\theta)=K_s\left(\frac{\theta-\theta_r}{\theta_s-\theta_r}\right)^{\frac{M}{N}} \tag{3}$$

where K and K_s are the unsaturated and saturated hydraulic conductivities respectively (cm/min), and M is a constant expressed as $M = 2 + 3N$.

The equation that describes the one dimensional vertical soil water flow (Richard's equation) with the corresponding initial and boundary conditions is:

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left(D(\theta)\frac{\partial\theta}{\partial z}\right)-\frac{\partial K(\theta)}{\partial z} \tag{4}$$

$$\theta(z,0)=\theta_i$$

$$\theta(0,t)=\theta_s$$

$$\theta(\infty,t)=\theta_i$$

where D is the diffusivity (cm^2 min^{-1}), θ_i is the initial water content (cm^3 cm^{-3}), z is the vertical distance (cm) with the vertical axis positive downward and t is the time (min).

The mathematical analysis made by Wang *et al*. (2003), concluded to the formulation of the following equations:

$$q=\frac{K_s}{\beta\ \ z_f}+K_s \tag{5}$$

$$\theta=\left(1-\frac{z}{z_f}\right)^{\alpha}\left(\theta_s-\theta_i\right)+\theta_i \quad \text{(where } \alpha=\frac{N}{M}\text{)} \tag{6}$$

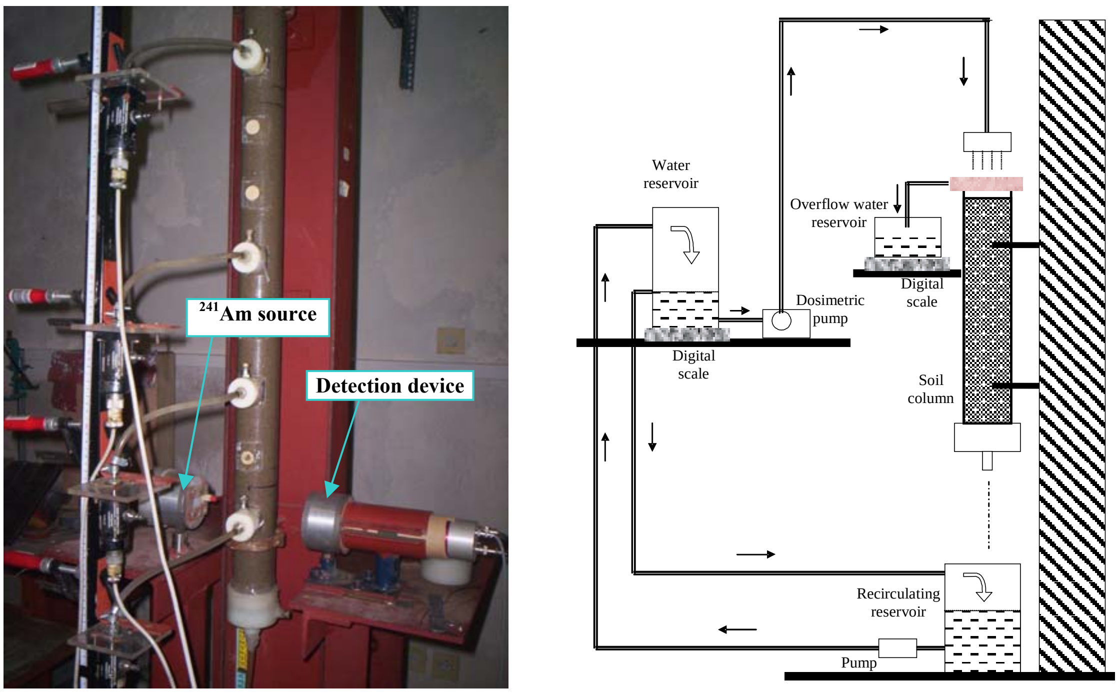
$$F = \frac{\theta_s - \theta_i}{1+\alpha} z_f \tag{7}$$

$$t = \frac{(\theta_s - \theta_i)}{(1+\alpha)K_s}\left[z_f - \frac{\ln(\beta\ z_f + 1)}{\beta} \right] \quad \text{(where } \beta = \frac{M}{g\ h_d} \text{ and } g \text{ parameter)} \tag{8}$$

Equations (5), (7) and (8) express Darcy's water flux (*q*), cumulative infiltration (*F*) and time (*t*) respectively, as a function of the wetting front distance (z_f), while equation (6) expresses soil water content (θ) as a function of both z and z_f. By obtaining experimentally the relationship between *F* and z_f, parameters α, β and K_s may be determined using equations (7) and (5). Therefore, $\theta(z)$ profiles may be calculated with equation (6).

3. APPLICATION

An infiltration experiment was carried out in the laboratory using a vertical cylindrical column made out of plexiglass, 100 cm long and 6 cm inside diameter. A sandy soil, with an initial water content of θ_i = 0.033 $cm^3\ cm^{-3}$, was air dried, graded between 0.425 and 0.600 mm in order to be homogeneous and packed into the column at a mean bulk density of 1.492±0.019 g cm^{-3}. A γ-ray device containing a 300 mCi ^{241}Americium source and a photomultiplier detector were set on a platform connected to a stepper motor, for the measurement of bulk density and soil water content (Reginato and van Babel, 1964) (figure 1a). Additionally, a dosimetric pump, an overflow construction and a re-circulation device were used for the achievement of constant head ponded water on the top of the soil column. Finally, two digital scales were necessary in order to measure the actual water volume entering the soil column during the experiment. Detailed description of the experimental setup for the constant head ponded water infiltration experiment is presented in figure 1b.



(a) (b)

Figure 1. a) Soil column with γ-ray device and b) Experimental setup

Using the above mentioned devices, soil water content in specific positions of the soil column (at distances z_1=14, z_2=45, z_3=65 and z_4=85 cm from the water inlet) and the time that wetting front arrived at these positions, were monitored temporally. Finally, the above mentioned devices (figure 1b) were also used during a separate experimental procedure, for the measurement of *K(θ)* and *Ψ(θ)*.

4. RESULTS

The constant head ponded water experiment lasted 65 min and the total water volume that entered in the soil sample was 650 cm^3. The final water content of this experiment was measured as θ_s=0.212 $cm^3\ cm^{-3}$ and the wetting front was detected in positions z_1, z_2, z_3 and z_4, on times t_1=6.9, t_2=19.7, t_3=28.5 and t_4=32 min respectively. Water content distribution at each one of the selected positions was represented graphically at figure 4. Finally, soil water content was obtained for times t_1, t_2 t_3 and t_4 as shown in figure 4.

Figure 2 shows a linear relationship between *F* and z_f for the experiment, from which parameter *α* was calculated utilizing equation (7) (*α* = 0.180). Furthermore, figure 3 shows a linear relationship between q and $1/z_f$, which permits the calculation of parameters β and K_s utilizing equation (5) (β=0.529 cm^{-1} and K_s =0.316 cm min^{-1}). The infiltration rate was calculated by the experimentally measured *F* values, using the formula *q=dF/dt*.

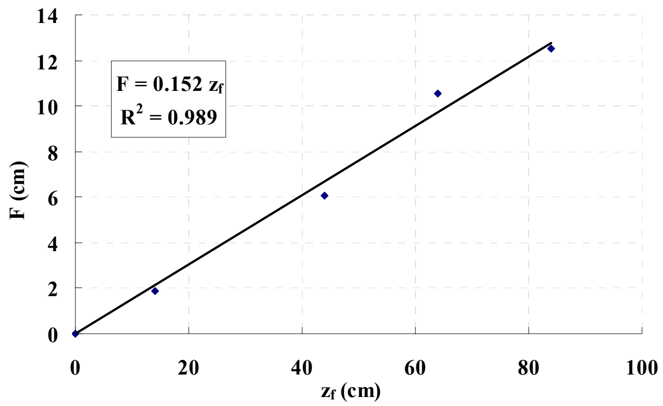


Figure 2. Relationship of cumulative infiltration F with z_f

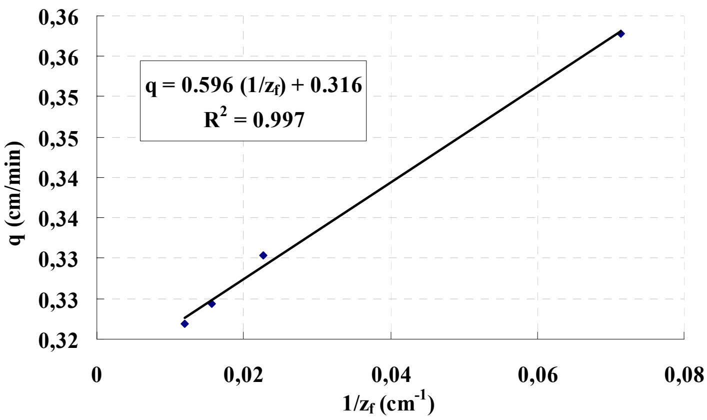


Figure 3. Relationship of water flux q with $1/z_f$

Soil water content profiles *θ(z)* where calculated, using equation (6). Figure 4 shows the measured and the calculated values of the water content profiles, for times t_1, t_2 t_3 and t_4 (points for measured and lines for calculated values).

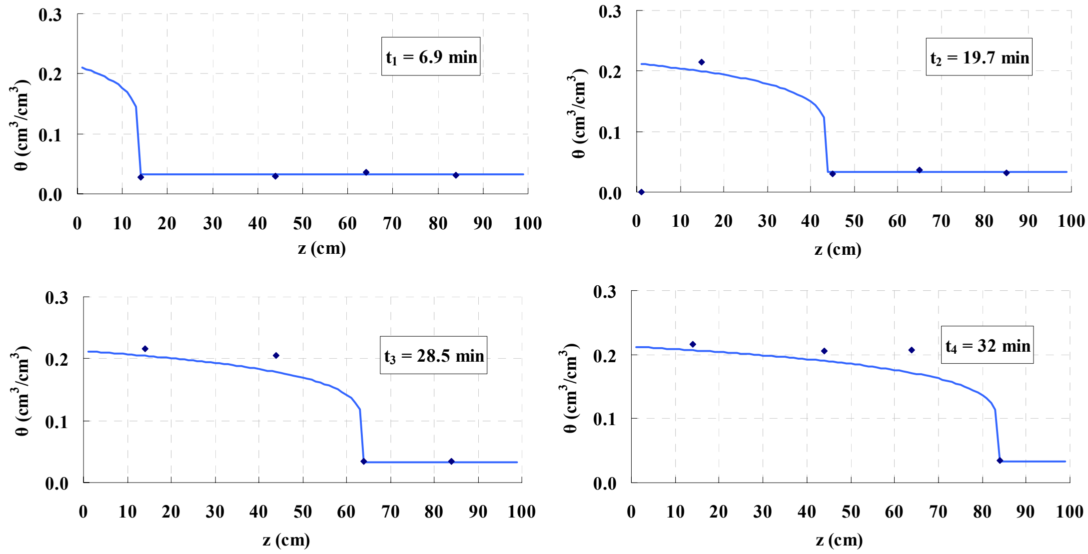


Figure 4. Soil water content distributions

In order to estimate the model's accuracy, measured infiltration time (t_m) with time calculated (t_c) from equation (8) was fitted through a linear function (figure 5). Furthermore, the water mass balance V(t) was estimated using equation (7) and the relationship $V=(\pi d^2/4)F$ for specific times (figure 6).

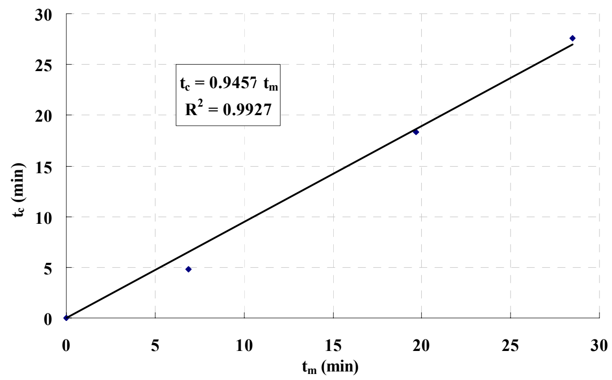


Figure 5. Relationship between calculated (points) and measured (line) infiltration times

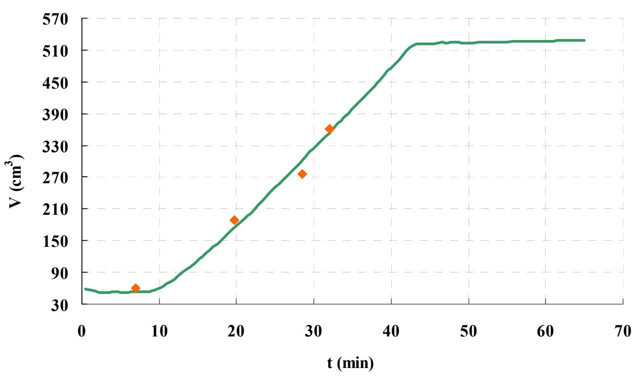


Figure 6. Calculated (points) and measured (line) water volume entered the soil column

Figure (7) shows the experimentally measured values of *K* and *θ* (points) that where fitted with equation (3) (line), utilizing the conjugate direction method (R^2=0.976). During the experiment for the measurement of K(θ) we were not able to measure K_s since final obtained water content was close but not equal to the saturation point (θ_s). K_s was obtained as a fitting parameter of equation (3) with a value equal to 1.299 cm min^{-1} and *M/N* = 2.143. During constant head ponded experiment the final water content reached 0.212 cm^3 cm^{-3} and the corresponding K(0.212) according to the Wang *et al*. (2003) model was calculated to be 0.316 cm min^{-1} and *M/N*=4.570. According to fitted equation (3), which was obtained from our hydraulic conductivity experiment, K for the same water content (0.212 cm^3 cm^{-3}) was calculated as 0.635 cm min^{-1}. Finally, figure (8) shows the soil water retention curves *Ψ(θ)* for a sequential procedure of drainage and wetting, that were also determined experimentally.

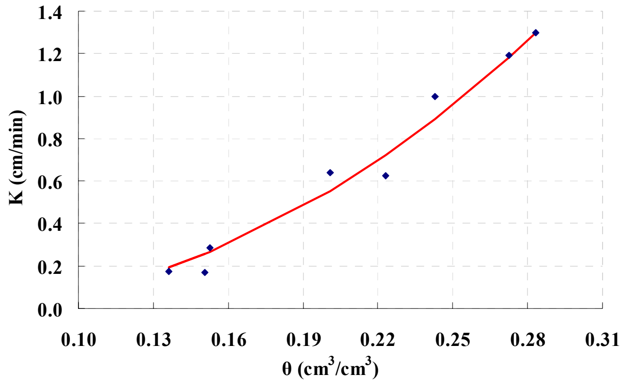


Figure 7. The relationship between K and θ

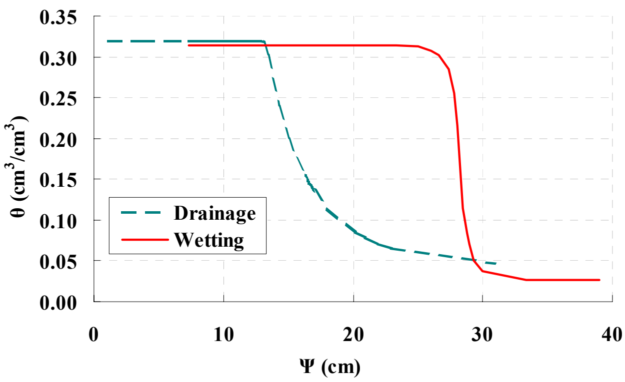


Figure 8. The soil water retention curves Ψ(θ)

5. CONCLUSIONS

The algebraic model developed by Wang *et al*. (2003) was simple to use, since the only data needed were the relationship between *F* and z_f, as well as θ_s and θ_i that were easily determined through the elaborated experiment. The infiltration process was quite rapid and for that reason it was difficult to obtain more experimental data and thus examine in more details the accuracy of the model. Nevertheless, the results show a fair agreement between calculated and measured values on soil water content profiles, hydraulic conductivity and on the water mass balance.

REFERENCES

Antonopoulos V. (2000) Modeling of soil water dynamics in an irrigated corn field using direct and pedotransfer functions for hydraulic properties, *Irrigation and Drainage Systems*, **14**, 325–342.

Arampatzis G. (2000) Experimental research on wetting and drainage to layered soils. Simulation with the method of finite volume. PhD thesis, Aristotle University of Thessaloniki.

Ashcroft G., Marsh D.D., Evans D.D., and Boersma L. (1962) Numerical method for solving the diffusion equation: 1. Horizontal flow in semi-infinite media, *Soil Sci. Soc. Proc.*, **26**,522–525.

Brooks R.H. and Corey A.T. (1964) Hydraulic properties of porous media, *Hydrology paper* No. 3*,* Colorado State Univ., Fort Collins.

Celia M.A., Bouloutas E.T. and Zarba R.L. (1990) A general mass-concervative numerical solution for the unsaturated flow equation, *Water Resour. Res.*, **26**(7),1483–1496.

Evangelides C., Tzimopoulos C. and Arampatzis G. (2005) Flux – saturation relationship for unsaturated horizontal flow, *Soil Sci.*, **170**(9), 671-679.

Parlange J-Y. (1971) Theory of water movement in soils: 2. One – dimensional infiltration, *Soil Science*, **111**(3), 170–174.

Parlange J-Y., Hogarth W.I., Barry D. A., Parlange M. B., Haverkamp R., Ross P. J., Steenhuis T. S., DiCarlo D. A. and Katul G. (1999) Analytical approximation to the solution of Richards' equation with applications to infiltration, ponding, and time compression approximation, *Advances in Water Resources*, **23**,189–194.

Philip J.R. (1973) On solving the unsaturated flow equation: 1 The flux-concentration relation, *Soil Sci.*, **116**(5), 328–335.

Reginato R.J. and van Bavel C.H.M. (1964) Soil water measurement with gamma attenuation, *Soil Sci. Soc. Am. Proc.*, **28**, 721–724.

Tzimopoulos C. (1978) Finite Elements Solution of Unsaturated Porous Media Flow, *Proceedings of the Second International Conference on Finite Elements In Water Resources*, Imperial College, London, Pentech Press **1**, 37–49.

Wang Q., Horton R. and Shao M. (2002) Horizontal infiltration method for determining Brooks-Corey model parameters, *Soil Sci. Soc. Am. J.*, **66**, 1733–1739.

Wang Quanjiu, Horton R. and Shao. M. (2003) Algebraic model for one-dimensional infiltration and soil water distribution, *Soil Science*, **168**(10), 671–676.