

## OPTIMIZATION OF THE FURROW IRRIGATION EFFICIENCY

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### ABSTRACT

Furrow irrigation is used for row crops. S.C.S., based on a great number of field experiments, under different circumstances and soil families, has developed design equations for furrow irrigation. These equations have been used for steady flow rate or for flow rate with one reduction when the water reaches the end of the furrow. They can also be used for a small period of time (time step) when the flow rate can assumed to be steady.

Many scientists, who dealt with furrow irrigation, try to improve the application efficiency by investigation the importance of the parameters that influence the furrow irrigation. Slope, furrow length, intake family have extensively been studied. Flow rate has been studied as steady or with a cutback of flow when water has advanced to furrow end. This article studies the influence of altering flow rate at furrow irrigation to application efficiency.

The steepest descent method is implemented for the optimisation at each time step flow rate. Steepest descent method evaluates the derivative of the function, which has to be optimized (application efficiency), to the parameter that is changing (flow rate). The derivative of the application efficiency to flow rate is calculated analytically. The optimized application efficiency is achieved when the derivative is zero, thus the value of flow rate is raised if the derivative is positive or is decreased if the derivative is negative.

When the flow rate at each time step is optimised, the process begins again because the optimisation for the flow rate of the initial steps didn't take into account the optimized flow rates of the later time steps. This process converges and estimates the optimised flow rate at each time step.

The results of the optimisation for two samples are alike. The maximum application efficiency is achieved by increasing the flow rate at the beginning of irrigation and before the water reaches furrow end the flow rate must be decreased to the minimum value of flow rate that fulfils instant infiltration.

**KEYWORDS:** Design equations of S.C.S., optimisation, altering flow rate.

### 1. INTRODUCTION

Furrow irrigation is a method of plant irrigation that is widely used because of its low cost in equipment and energy. Furrow irrigation has low application efficiency, while newer irrigation systems have much higher application efficiency.

Several parameters are evaluated at furrow irrigation design and our aim is to choose the value of the changeable parameters so that the maximum application efficiency of irrigation will be achieved. S.C.S. developed equations for the calculation of the advance time, the design time, the wetted perimeter, the application efficiency when values of flow rate, length, slope, Manning roughness coefficient and intake family of furrow are known.

Most of the scientists (Hart *et al.*, 1983; Papazafeirou, 1984; Panoras *et al.*, 1996, Panoras, 1988, Papamichail and Papadimos, 1995, 1996) who have dealt with furrow irrigation, have assumed that the changeable parameters are the flow rate and the length of the furrow. The influence of the length has been studied extensively from a number of scientists. It is known that optimization of uniformity and minimization of surface runoff, which leads to greater application efficiency, can be achieved by using an altered flow rate. The only case of altering flow rate, which has been studied, is irrigation with one cutback of flow, when water has advanced to furrow end.

In this article the application efficiency is optimized by using altering flow rate.

## 2. S.C.S. DESIGN EQUATIONS FOR FURROW IRRIGATION

S.C.S. has developed equations for furrow irrigation design by classifying soil to intake families, has set constants unique for each intake family given in tables or equations (Hart *et al.*, 1983, Papamichail and Papadimos, 1995, Papamichail and Papadimos, 1996).

The cumulative infiltration is expressed as the equivalent intake depth over the furrow spacing and unit length:

$$F_n = (aT^b + c) \frac{P}{W} \quad (1)$$

where  $F_n$  is equivalent intake depth in mm,  $T$  is time in min,  $a$ ,  $b$ , and  $c$  are intake family coefficients,  $w$  is the furrow spacing,  $P$  wetted perimeter per unit length, given by the empirical relationship

$$P = 0.265 \left( \frac{nQ}{S^{1/2}} \right)^{0.425} + 0.227 \quad (2)$$

where  $Q$  is the flow rate ( $l\ s^{-1}$ ),  $S$  is the slope or hydraulic gradient ( $m\ m^{-1}$ ), and  $n$  is the Manning roughness coefficient.

The time for water to advance to a specific point along the furrow depends on the distance, flow rate and slope:

$$T_t = \frac{x}{f} e^{gx/QS^{1/2}} \quad (3)$$

where  $T_t$  is the advance time (min),  $x$  is the distance (m) from upper end of the furrow to point  $x$  (maximum value of  $x$  is  $L$ , the field length),  $f$  and  $g$  are advance coefficients given in tables or equations.

The opportunity time required for intake of the selected net application depth,  $F_n$ , can be estimated from the solution of equation 1 in the form

$$T_n = \left[ \left( d_n \frac{W}{P} - c \right) / a \right]^{1/b} \quad (4)$$

Small dimensions of furrows and erosion demand irrigation with low flow rate. Thus, the remained water infiltrates very fast and the recession time is assumed zero, for gradient and open furrows.

Irrigation stops when net application depth is infiltrated at the end of the furrow. With the above assumption, the irrigation time  $T$  is the sum of the advance time to the end of the furrow and the opportunity time:

$$T_a = T_n + T_t = T_n + \frac{L}{f} e^{gL/QS^{1/2}} \quad (5)$$

where  $T_a$ ,  $T_t$ ,  $T_n$ , are in min,  $L$  is the length of the furrow in m.

The gross water application ( $F_g$ ) is:

$$f_g = \frac{60 QT_a}{WL} \quad (6)$$

where  $F_g$  is in mm.

The application efficiency  $E_f$ , expressed in percent is:

$$E_f = \frac{100 f_g}{f_y} \quad (7)$$

The above procedure is referred when the flow rate is constant during the irrigation. The flow rate may be reduced improving the application efficiency. Many designers are reducing the initial flow rate to the half at the time the initial flow has advanced to the end of the open end furrow. Advance time is computed for the equation (3), using the initial flow rate  $Q$  and  $X=L$ , where  $L$  is the length of the furrow. The opportunity time for intake of the desired net application  $F_n$  is calculated from equation (4) where the adjusted wetted perimeter is determined for the reduced flow rate  $Q/2$ . The total time for irrigation is the sum of  $T_t$  and  $T_a$ . The new gross application is:

$$f_y = \frac{60}{WL} \left( QT_t + \frac{Q}{2} T_n \right) \quad (8)$$

### 3. APPLICATION OF THE ALTERING FLOW RATE AT THE S.C.S. EQUATIONS

Assuming that for a small period of time (time step) flow rate is steady, S.C.S. equations can be used for that period of time. The estimation of the advance and opportunity time is alike when the altering flow rate is used. The distance that the water will advance or the quantity of water that will infiltrate, is calculated at each time step. Both equations that describe infiltration and the advance of the water are nonlinear to the time. Each time step does not cause the same alteration to the distance and the cumulative infiltration. Thus, the alteration to the distance and cumulative infiltration has to be calculated at each time step. That alteration is caused by the alteration to time equal to the time step ( $Dt$ ), for a specific flow rate.

Regarding the calculation of the advance time, firstly, the distance that the water has advanced with a constant flow rate is calculated for a specific time ( $Dt, 2Dt, \dots$ ). Equation (3) is used for that calculation, which needs a numerical method to be solved for the distance e.g. the midpoint method was used. Next, the advance time for that distance with the flow rate of the new time step is calculated and the process begins again. The process ends when the calculated distance is bigger than the length of the furrow. The advance time is computed from the number of time steps that had been done until the water reaches the end of the furrow. One calculation phase of this process is plotted in Figure 1. For time  $T_1$ , the corresponding flow rate is  $Q_1$  and the distance  $X$  is calculated. Next, a new advance curve is calculated because flow rate changes. Time  $T_2$  is the time required for the water to advance the same distance with  $Q_2$  flow rate. The time step  $Dt$  is added to  $T_2$  and a new calculation phase starts.

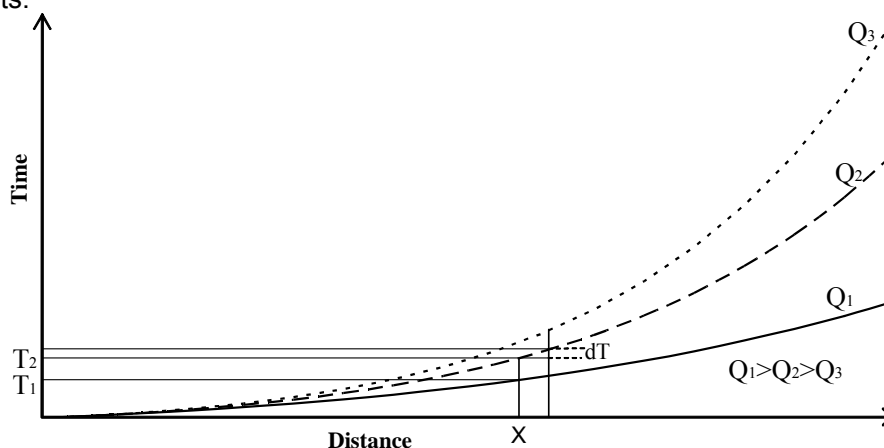


Figure 1. Distance to time for different flow rates

The same process is implemented for the calculation of the opportunity time. At the beginning, for a specific time ( $Dt, 2Dt, \dots$ ), the wetted perimeter is calculated taking into account the flow rate of that specific time and then the cumulative infiltration is calculated from equation (1). The opportunity time is calculated from equation (4), for the same quantity of water to infiltrate, but with the wetted perimeter that has the flow rate of the new time step.

The time step is added to that time and the process begins again. The process ends when the cumulative infiltration becomes bigger than the net application depth. The opportunity time is computed from the number of time steps required for the net opportunity depth to infiltrate at the bottom edge of the furrow. One calculation step of this process is plotted in Figure 2. For time  $T_1$ , the flow rate is  $Q_1$  and the cumulative infiltration  $I$  is calculated. Then flow rate change, therefore a new curve of cumulative infiltration is formed. The calculated time  $T_2$  is the time required for the same amount of water to infiltrate, if flow rate is  $Q_2$ . The time step  $Dt$  is added to  $T_2$  and a new calculation step starts (Ampas 1998, Ampas and Baltas, 2007).

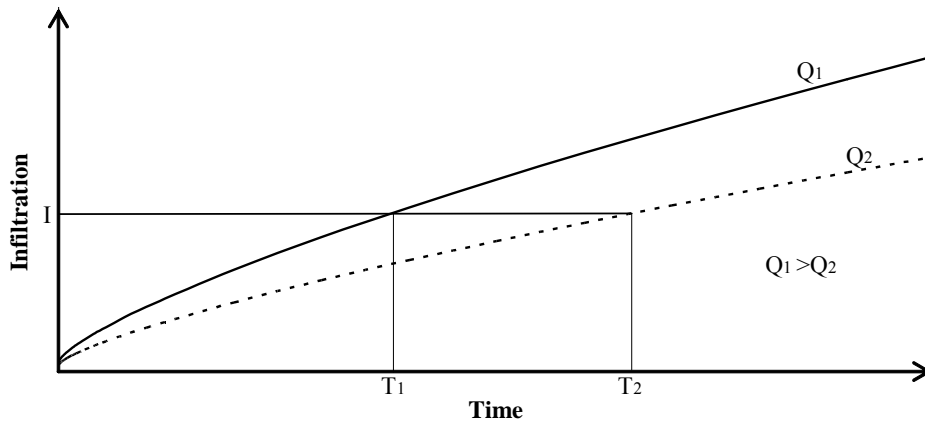


Figure 2. Cumulative infiltration to time for different flow rates

The gross application depth is calculated as the integral of the flow rate to the time, with the relationship:

$$d_g = \frac{60}{WL} \sum Q_i * Dt \quad (9)$$

where  $Q_i$  is the flow rate at the  $i$  time step,  $Dt$  is the time step in min.

#### 4. MAXIMISATION OF THE APPLICATION EFFICIENCY

The maximisation of the application efficiency to the flow rate is a non linear, constrained optimisation problem. The parameters for the optimisation are as many as the time steps that the irrigation takes place. We have the equation  $Ef(Q_1, Q_2, \dots)$  and our aim is the calculation of the maximum  $Ef(Q_1, Q_2, \dots)$ .

Constraints for the optimisation are the maximum and the minimum value of flow rate. The minimum value of the flow rate must fulfil the instant infiltration for all the length of the furrow. The calculation of the minimum value of the flow rate is based on the instant infiltration. However, instant infiltration does not have the same value for all the length of the furrow, because opportunity time differs at each point and depends on the advance time. Thus, the length has to be divided into  $N$  equal parts with  $Dx$  length, to each one of which the infiltration is calculated and then the minimum flow rate is calculated from the relationship:

$$Q_{\min} = \sum_{i=1}^N \frac{I_i * P * Dx}{60} \quad (10)$$

where  $Q_{\min}$  is the minimum flow rate in  $l s^{-1}$ ,  $I$  is the instant infiltration to the  $i$  part of the furrow in  $mm min^{-1}$ ,  $P$  is the wetted perimeter in m. The value of the minimum flow rate can be computed with iterations because wetted perimeter is a function of the flow rate.

The maximum value of the flow rate without causing erosion can be computed with the empirical relationship (Hart *et al.*, 1983):

$$Q_{\max} = 0.63/S \quad (11)$$

where  $Q_{\max}$  is the maximum value of the flow rate in  $l s^{-1}$  and  $S$  is the furrow slope  $m m^{-1}$ .

With the solution of the optimization problem the constant values at each one time step are determined i.e.  $Q_1, Q_2, \dots$ , for the maximum application efficiency of the irrigation.

To maximise the application efficiency, with an optimisation method, the flow rate that maximises the application efficiency when all the other values of the flow rate are constants, is calculated at each time step. First we evaluate the optimum values of the first time steps flow rate then the optimum values of the second time steps flow rate and the process stops when the irrigation is over. At this process the initial time steps flow rates do not use the optimum values of the flow rate of the afterwards time steps. Thus new calculation must be done to compute the optimum values for each time step. The new values of flow rate converge to the optimum values of the flow rate. This process avoids large matrixes.

The estimation of the optimum flow rate at each time step is a non linear optimization problem and needs an iterative – converged method, like the steepest descent method (Press et al, 1992, Georgiou and Vasiliou, 1993, Ampas, 1996). According to the method, the slope of the curve of application efficiency versus the flow rate  $Q_i$  is determined at the point of estimation of the flow rate  $Q_i^K$  ( $K$  estimation of the flow rate at the  $i$  time step). The slope, which mathematically is expressed from the derivative, is computed numerically. To the direction that the application efficiency is maximized, the flow rate  $Q_i^K$  is changed. We change the flow rate so that the new application efficiency is bigger than the previous one. With the new estimation of the flow rate  $Q_i^{K+1}$ , we compute the slope, e.t.c., until the value of the flow rate maximizes the application efficiency. The amount of change to the value of the flow rate  $Q_i^K$  is very important for the success of the method. If the change is bigger than it should be, then the application efficiency might be lower and if the change is smaller than it should be, it will result in a large number of iteration.

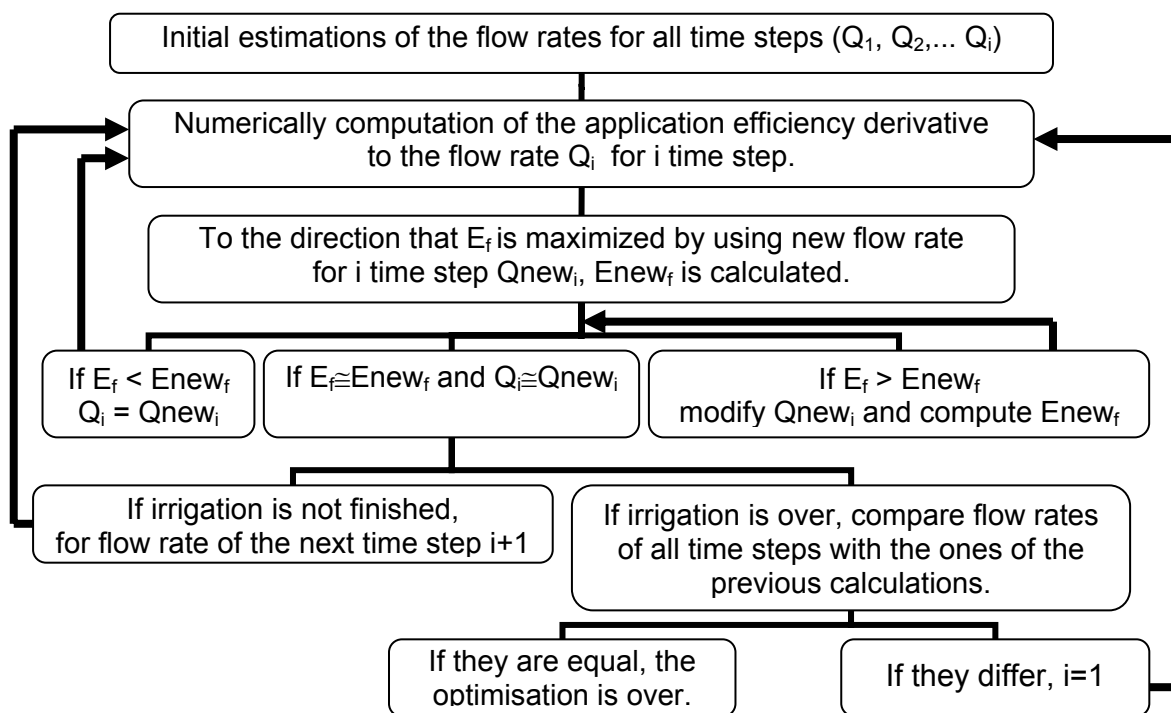


Figure 3. Flow chart for the optimization

The algorithm of the method is given below:

1. Initial estimations of the flow rates are made for all time steps ( $Q_1, Q_2, \dots, Q_i$ ).
2. For the flow rate at  $i$  time step, the derivative of the application efficiency to the flow rate  $Q_i$  is computed numerically.
3. To the direction that  $E_f$  is maximized by using new flow rate for  $i$  time step  $Q_{new_i}$ , the new application efficiency  $E_{new_f}$  is calculated.
4. If  $E_f \approx E_{new_f}$  and  $Q_i \approx Q_{new_i}$

- a) If irrigation is not finished, for flow rate of the next time step  $i+1$ , we go to 2.
  - b) If irrigation is over, the flow rates of all time steps are compared with the ones of the previous calculations. If they differ for  $i=1$ , we go to step 2, if they are approximately equal, then the optimisation is over.
5. If  $E_f > E_{new_f}$ , according to 3, we modify  $Q_{new_i}$  and compute a new application efficiency  $E_{new_f}$  we go to 4.
  6. If  $E_f < E_{new_f}$ , we have  $Q_i = Q_{new_i}$  and go to 2.

This optimization method, as it is applied for the estimation of one optimum flow rate  $Q_i$  is the one – dimension expression of the steepest descent method. The application of the full form of the steepest descent method for the estimation at the same time, the optimum flow rates at all time steps demands large matrixes, which are avoided with the modification of the method as it is implemented here.

## 5. SAMPLES

Two applications from the literature were selected for the evaluation of the flow rate that maximizes the application efficiency. The first one is from Hart *et al.* (1983) and the second one is from Papazafeiriou (1984). The results are shown in tables and charts.

### 5.1. First Sample

Intake family of the soil	$I_f = 0,3$	Roughness coefficient	$n = 0.04$
Net depth of application	$d_n = 75 \text{ mm}$	Furrow spacing	$w = 0.75 \text{ m}$
Furrow length	$L = 275 \text{ m}$	Slope	$S = 0.004 \text{ m m}^{-1}$

Table 1. Results for the three different irrigations

	<b>P (m)</b>	<b><math>T_t</math> (min)</b>	<b><math>T_n</math> (min)</b>	<b><math>T_a</math> (min)</b>	<b><math>d_t</math> (mm)</b>	<b><math>E_f</math> (%)</b>
Constant flow rate	0.40	143.6	999	1143	200	37.5
One reduction of flow rate	0.36	143.6	1165	1309	127	59
Altering flow rate	-	100.3	1270.5	1370.8	99.8	75.1

At Table 1, in the first row, we have the results when we irrigate with stable flow rate, in the second row we have the results when we reduce the flow rate to half when the water reaches the bottom end of the furrow, in the third we use altering flow rate.

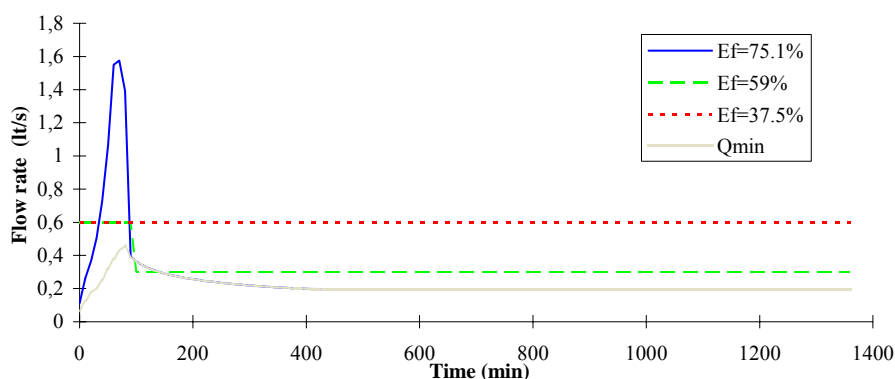


Figure 4. Flow rate of irrigation to time for sample 1

The results when we use altering flow rate are given from a computer program, which was designed for this aim, using the S.C.S. equations as presented above for altering flow rate. The time step was 10 min, the space step for the minimum flow rate estimation was 1m. The program was as a subroutine to a program for the optimization using the steepest descent method.

There are 138 time steps. Flow rate versus time is shown in Figure 4, as well as the plots

for the two other irrigations and the minimum flow rate.

5 circles of calculations were made for the optimization. The optimum flow rate at each time step was calculated by 7 iterations at the initial circles and only 1 or 2 iterations were necessary at the latest circles as the procedure converged.

Using smaller time step ( $Dt = 5$  min), the curve of the flow rate versus time didn't change and there was a small increase of the application efficiency at 0.05 %.

## 5.2. Second Sample

Intake family of the soil	$I_f = 0,5$	Roughness coefficient	$n = 0.04$ m
Net depth of application	$d_n = 80$ mm	Furrow spacing	$w = 1.00$ m
Furrow length	$L = 300$ m	Slope	$S = 0.005$ m m <sup>-1</sup>

Table 2. Results for the three different irrigations

	P (m)	T <sub>t</sub> (min)	T <sub>n</sub> (min)	T <sub>a</sub> (min)	d <sub>t</sub> (mm)	E <sub>f</sub> (%)
Constant flow rate	0.435	111.3	796.3	907.6	181.5	44.1
One reduction of flow rate	0.382	111.3	953.5	1064.8	117.6	68.0
Altering flow rate	-	110	998.1	1108.1	101.5	78.8

Table 2 is similar to Table 1 for the second sample. Also, Figure 5 is similar to Figure 2. 9 circles of calculations were made for this optimization. The number of each time step optimum flow rate is almost the same as the previous sample.

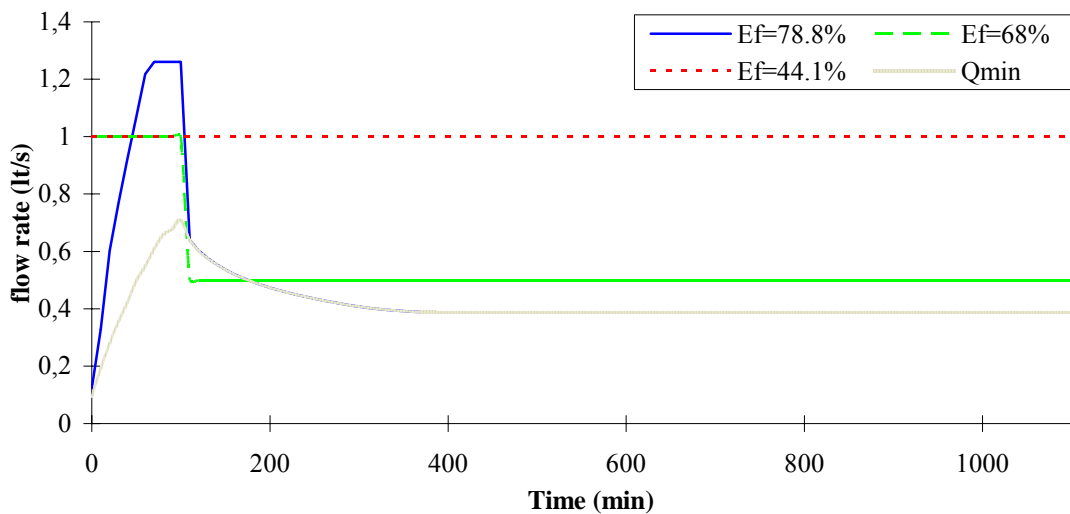


Figure 5. Flow rate of irrigation to time for sample 2

## 6. DISCUSSION

The results were alike for both samples. The procedure of altering the flow rate resulted in application efficiency much higher than that of the constant flow rate. The increase of the application efficiency was up to 90 % of the efficiency with constant flow rate, and it is quite higher if the flow rate is reduced at half when the water reaches the bottom end of the furrow.

The curve of the flow rate versus time is alike at both samples. At the beginning, a relatively low flow rate is applied, which is increased until it takes its maximum value. When the water reaches the bottom end, the flow rate is decreased until it takes its minimum value. The minimum value of the flow rate fulfils the instant infiltration (basic infiltration) for all the length of the furrow, is one of the constraints of the optimization.

There are two reasons for the increase of the application efficiency. The first one is the decrease of surface runoff and the second is the increase of application uniformity.

Surface runoff is decreased because the applied flow rate is calculated just to fulfil the instant infiltration by applying the minimum flow rate ( $Q_{\min}$ ).

The application uniformity with altering flow rate is higher due to the way that flow rate changes with time. Small value of the flow rate at the beginning of the irrigation cause small value of wetted perimeter, therefore small value of the cumulative infiltration. The increase of the infiltration after some time causes big wetted perimeter. At the top of the furrow, the cumulative infiltration is relatively big, so the value of the instant infiltration has decreased, therefore the quantity of the water that will infiltrate is small. At the middle and at the end of the furrow, the big wetted perimeter and the big instant infiltration cause a big quantity of the water to infiltrate. Thus, the distribution of water is more uniform and the application efficiency increases.

Comparing the two samples that have different irrigation factors we can say:

- Slope is an important factor of the application efficiency as it restricts high values of flow rate and simultaneous higher values of slope makes the water to advance at furrow's bottom end faster and therefore it achieves higher uniformity.
- Intake family is a very important factor for the application efficiency as it determines the instant infiltration. As mentioned above knowledge of instant infiltration reduces surface runoff, but also instant infiltration influences application uniformity because high values of infiltration cause long advance time and the water reaches furrow's bottom end later, therefore application uniformity and application efficiency are decreased.

Irrigation with one reduction of flow rate when water reaches furrow's bottom end is not right because it does not fulfil the instant infiltration for all the length of the furrow. As it can be seen in both figures 4 and 5 the curve of irrigation with reduction of flow rate is below the curve of the minimum flow rate ( $Q_{\min}$ ).

## 7. CONCLUSIONS

The aim of this paper is to maximize the application efficiency of furrow irrigation. For the design of the furrow irrigation, the equations of S.C.S. were properly modified, so that they can be used for flow rate constant for a small period of time. Several methods were tested for the optimization. The most effective was the steepest descent method for the computation of the optimum value of the flow rate at each time step.

The flow rate of furrow irrigation must start from a small value which is increasing until it reaches the maximum allowed value. Flow rate then is decreased just to fulfil the instant infiltration. Further research must be done that will examine the altering of the flow rate for different soil types and different slopes of the cropped field.

Agriculture is using the biggest amounts of water, thus higher application efficiency can save very big amounts of fresh water. This economy can be achieved with the practice of the theory. It is a quite common practice for the farmers, to reduce the flow rate at the furrow when water reaches the furrow end by moving higher the pipes. It is very easy to irrigate the next furrow instead of reducing the flow rate. Thus the next furrow will start its irrigation with a small flow rate which can be increased later when the previous furrow will stop its irrigation. Of course, small time steps for the altering of flow rate are impossible without an automatic control. Practically is not difficult to alter 3 or 4 times the flow rate of irrigation, this will increase the application efficiency.

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