Global NEST Journal, Vol 14, No 3, pp 335-343, 2012

SENSITIVITY ANALYSIS OF DIFFERENT EVAPOTRANSPIRATION METHODS USING A NEW SENSITIVITY COEFFICIENT

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Received: 20/05/12
Accepted: 27/07/12
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ABSTRACT

The estimation of evapotranspiration is essential in water resources management. Among a group of methods, the Penman–Monteith has been commonly applied to calculate reference evapotranspiration as this method has been also recommended by the Food and Agriculture Organization of the U.N. (FAO). Other methods widely used are: the FAO 24 Penman, the modified Blaney and Criddle, the FAO 24 Makkink, and the Hargreaves.

Sensitivity analysis is required to gain a better understanding of the meteorological systems; particularly to indicate the physical meaning of each meteorological parameter used in the estimation of the reference evapotranspiration. Several dimensionless sensitivity coefficients have been proposed, based on the partial derivative of the dependent variable (reference evapotranspiration) to the independent variables (meteorological variables).

In this paper, a new sensitivity coefficient is proposed to drive sensitivity analysis of the evapotranspiration methods. The new sensitivity coefficient uses the partial derivative and the standard deviation of each independent variable. The meteorological variables, whose influence has been examined, are all the necessary meteorological parameters for the calculation of reference evapotranspiration, such as temperature, solar radiation, wind speed and relative humidity for each method. Data from the automatic meteorological station of Aminteo in the Prefecture of Florina, Western Macedonia, were used. The sensitivity coefficients were calculated for each month, year and irrigation period. The comparison of the sensitivity coefficients is performed for the month of water peak demand (July), the irrigation period and the year for each evapotranspiration method.

Results show that the influence of the variables to evapotranspiration is not the same for each period, and also the order that the variables influence evapotranspiration is changing. A comparison between the five evapotranspiration methods shows that solar radiation and temperature are the main parameters that affect evapotranspiration, while relative humidity and wind speed are not so important for the calculation of evapotranspiration.

KEYWORDS: sensitivity coefficient, reference evapotranspiration, evapotranspiration method, meteorological parameters.

1. INTRODUCTION

Evapotranspiration is an important component of the hydrologic cycle as it can significantly affect the water budget of the natural (i.e. approximately 62% of all precipitation falling on land is evapotranspirated. Consequently, its accurate estimation is essential for, among others, water availability, plant growth, irrigations efficiency, reservoir operation,,and water resources management. Several empirical methods have been developed to derive evapotranspiration estimates. Among others, the Penman–Monteith method is recommended by the Food and

Agriculture Organization of the U.N. (FAO) as the sole method to calculate reference evapotranspiration, wherever the required input data (i.e. temperature, relative humidity, solar radiation, wind speed) are available (e.g., Allen *et al*., 1998; Ampas, 2010). Other methods widely used are the FAO 24 Penman method, the FAO 24 Blaney and Criddle method (Doorenbos and Pruitt, 1977), the FAO 24 Makkink method, and the Hargreaves method.

Sensitivity analysis has been an important stage on the evaluation of environmental models; however, current research urges the need to assess the physical meaning of model parameters and their relative influence on the meteorological variables. By definition sensitivity analysis studies the impact of the change of one parameter to another (McCuen, 1973).

Several studies have assessed the parameter sensitivity to estimated evapotranspiration using sensitivity coefficients which were calculated for several independent variables as meteorological parameters, physiological parameters, and climatic conditions. Comparison of sensitivity coefficients has showed the relative importance of each variable. Saxton (1975) conclude that the most important variable for the calculation of ET_o, during summer is solar radiation, whilst in autumn and spring the most important variable is the aerodynamic variable. Coleman and DeCoursey (1976) conclude that the most important parameter at the annual scale is relative humidity; during summer both temperature and solar radiation are the most important variables, whereas relative humidity is more important during winter. They also conclude that wind speed has very small importance at the annual scale. Babajimopoulos *et al.* (1992) conclude that temperature and solar radiation are the most important variables in the summer, whereas the most important parameter in the winter is relative humidity (wind speed has very small importance). Gong *et al.* (2006) evaluated sensitivity coefficients for the Yangtze River basin and indicated their large spatial variability. Irmak *et al.* (2006) evaluated sensitivity coefficients for areas under different climatic characteristics. Results showed large spatial variability, and the authors concluded that for areas with strong and dry winds wind speed was the most important variable.

In this paper, we assess parameter sensitivity to the estimated evapotranspiration based on five methods (FAO 24 Penman, FAO 56 Penman-Monteith, FAO 24 Blaney-Criddle, FAO 24 Makkink, and Hargreaves method) and evaluate the impact of the change of the measured meteorological variables to the estimated reference evapotranspiration. Finally, we compare the relative influence of each meteorological parameter to reference evapotranspiration. The sensitivity analysis is based on a new sensitivity coefficient designed for the comparison of the influence of the independent parameters and uses standard deviation. To address the above listed points, the paper is organised as follows. The evapotranspiration methods and the sensitivity analysis are introduced in Section 2. In Section 3, the study area, instruments and data are introduced. Section 4 presents results and finally, Section 5 states the conclusions.

2. METHODOLOGY

2.1. Reference evapotranspiration

The evapotranspiration rate from a reference surface, not short of water, is called reference crop evapotranspiration or reference evapotranspiration (ETo) reference surface is a hypothetical grass reference crop with specific characteristics. ETo expresses the evaporating power of the atmosphere at a specific location and time of the year and does not consider the crop and soil characteristics (Allen *et al.*, 1998, A.S.C.E., 2005).

This index was been introduced to study the evaporative demand of the atmosphere independently of crop type, crop development and management practices, hence it is only affected by meteorological properties (i.e. temperature, Rel. humidity, wind speed, Solar Radiation).

Numerous empirical methods have been developed over the last 50 years to estimate evapotranspiration using different climatic variables. However, relationships were often subject to rigorous local calibrations and proved to have limited global validity (Allen *et al.*, 1998).

FAO 24 Penman method

Doorenbos and Pruitt (1977) modified the original Penman method (now named FAO 24 Penman method) introducing a wind function and a multiplier (c) which depends on the local climatic conditions, to downscale the a priori estimation. The method estimates the evapotranspiration from grass and is given by:

$$\mathrm{ET_o} = c\left[\frac{\Delta}{\Delta+\gamma}(R_n - G) + \frac{\gamma}{\Delta+\gamma}2.7(1.0+0.864u_2)(e_s - e_a)\right] \tag{1}$$

where ET_o is the reference evapotranspiration (mm), R_n is the net Radiation (MJ m^{-2} d^{-1}), Δ is the slope of saturation vapour pressure curve (kPa $°C^{-1}$), e_s is the mean saturation vapour pressure (kPa), e_a is the actual vapour pressure (kPa), u_2 is the mean wind speed at height 2m (m s^{-1}), γ is the psychrometric constant (kPa $°C^{-1}$), and G is the soil heat flux (MJ m^{-2} d^{-1}).

FAO 56 Penman – Monteith method

The most widely applied method to estimate evapotranspiration is the Penman-Monteith combination model. This method takes into account both meteorological and physiological crop variables (Allen *et al.*, 1998; see Chapter 3 of the FAO paper 56). For 24-hour calculations of ET_o, from daily data the FAO Penman-Monteith is given by:

$$\mathrm{ET_o} = \frac{0.408\cdot\Delta\cdot\left(\mathrm{R_n}-\mathrm{G}\right)+\gamma\cdot\frac{900}{\mathrm{T}+273}\cdot\mathrm{u_2}\cdot\left(\mathrm{e_s}-\mathrm{e_a}\right)}{\Delta+\gamma\cdot\left(1+0.34\cdot\mathrm{u_2}\right)} \tag{2}$$

where ET_o, R_n, Δ, e_s, e_a, u_2, γ, and G as above, T is the mean daily temperature.

FAO 24 Blaney - Criddle method

Doorenbos and Pruitt (1977) modified the initial equation of Blaney and Criddle and gave a new form that can be used for the reference crop evapotranspiration. The method is based on the linear relation between the factor $\left[(0.46\mathrm{T}+8.13)\ \mathrm{p}\right]$ from the original Blaney and Criddle method and the measured value of evapotranspiration. In a recent study, Ampas and Baltas (2007) estimated factor b precisely. The new equation has the form:

$$\mathrm{ET_o} = \left[0.0043\mathrm{RH_{min}} - \frac{\mathrm{n}}{\mathrm{N}} - 1.41\right] + \mathrm{b}\left[(0.46\mathrm{T}+8.13)\ \mathrm{p}\right] \tag{3}$$

where b is a function of minimum Relative Humidity, relative sunlight and wind speed, T is the mean daily temperature (^{o}C), RH_{min} is the minimum relative humidity of the atmosphere (%), n/N is the relative sunlight, n is the actual sunshine (hr), p is the day-hours percentage of the year hours.

FAO 24 Makkink method

The methodology followed by Makkink (1957) assumes that most of the evapotranspiration takes place due to the energy from radiation and temperature difference (hence energy) between the air above the surface and the surface (see Doorenbos and Pruitt, 1977). Both energy sources are associated and can be expressed from solar radiation. The equation is given by:

$$\mathrm{ET_o} = a + b\left(\frac{\Delta}{\Delta+\gamma}\right)R_s \tag{4}$$

Where Δ and γ as adove, a is a coefficient (mm d^{-1}), b is a function of mean Relative Humidity (%) and wind speed (m s^{-1}), R_s is solar radiation (mm d^{-1}).

Hargreaves method

Hargreaves (1975) developed an empirical relation for reference evapotranspiration. The Hargreaves method is proposed by FAO when there aren't enough data available or when data are not reliable and uses only temperature. This method is very simple, practical and gives acceptable estimates. The Hargreaves equation is given by:

$$ET_o = 0.0023(T_{mean} + 17.8)(T_{max} - T_{min})^{0.5}\ R_a \tag{5}$$

where T_{mean}, T_{max}, T_{min} T is the mean, maximum and minimum temperature respectively (^{o}C), and R_a is the extraterrestrial radiation (MJ m^{-2} d^{-1}).

2.2. Sensitivity analysis

By definition, sensitivity analysis investigates the effect of change of one factor on another (McCuen, 1973). The change of reference evapotranspiration to the change of a meteorological variable, when it tends to zero, is the partial derivative of reference evapotranspiration to this variable. A number of sensitivity coefficients can be defined based on dimensionless values of the reference evapotranspiration change for different purposes of sensitivity analysis (McCuen, 1974, Saxton, 1975, Beven, 1979, Gong *et al.*, 2006). The dimensionless values of sensitivity coefficients for different meteorological parameters allow the comparison between them. Saxton (1975) defined dimensionless sensitivity coefficients for each meteorological variable based on:

$$Ks_p = \frac{\partial M}{\partial p}\frac{p}{M} \tag{6}$$

where p is the examined independent variable or parameter and M is the modelled value.

This coefficient shows the percentage of change in evapotranspiration caused by the percentage change of a meteorological variable. The calculation of the partial derivative of reference evapotranspiration to a variable depends on all the meteorological variables and its value depends on them. However, Equation 6 is sensitive to the magnitudes of reference evapotranspiration and p. In particular, the relative sensitivity coefficient Ks_p may not be a good indication of the significance of the variable if either: 1) the value of reference evapotranspiration or the value of the parameter tends to zero independently, or 2) the range of values taken by p is small in relation to its magnitude (Beven, 1979).

Coleman and DeCoursey (1976) provided a more meaningful coefficient when comparing variables some of which may have a range in variability quite different from their mean value; hence the bias caused by the method of measurement is eliminated. The coefficient is given by:

$$Ks_p = \frac{\partial M}{\partial p}\frac{p - p_{min}}{M} \tag{7}$$

where p_{min} is the minimum observed value of the independent variable.

Babajimopoulos *et al.* (1992) estimated the influence of the meteorological variables to evapotranspiration changing by 10, 20 and 30% the meteorological variables and assessing its impact on the calculated evapotranspiration. However, in this case the variation of a parameter could significantly influence the sensitivity of the parameters to the model. More recently, Ampas (2010) proposed the use of standard deviation and presented a new sensitivity coefficient:

$$Ks_p = \frac{\partial M}{\partial p}\frac{\sigma_p}{M} \tag{8}$$

where σ_p is the standard deviation of the meteorological variable.

It is important to note the advantages of Equation 8 to the previous approaches.

- Standard deviation can't be zero.
- The coefficient is not influenced by the units.
- Standard deviation expresses the entire data set.
- The minimum value depends on the magnitude of the time series.
- The range width depends on both minimum and maximum values.
- Some meteorological parameters, as Relative Humidity and wind speed are limited, RH from 0-100% and u_2 is positive (> 0 m s^{-1}).
- This sensitivity coefficient shows the alteration to the model caused by the usual change of the parameter. In contrast, Equation 6 represents the ability of each parameter to change the model.

3. STUDY AREA AND INSTRUMENTS

The 1924 km^2 study area is located at the prefecture of Florina, Western Macedonia, Greece. Ten automatic meteorological stations (AMS) operate within the study area and consist of sensors, a data logger, a communication system and a power supply system. The established sensors measure temperature, relative humidity, wind speed, wind direction, rain, solar radiation, and sunshine duration every 10 seconds. The data logger records the data at an hourly resolution. The communication with the stations can be achieved using modems, UHF radio modem, G.S.M. modems or G.P.R.S modems whereas data can be downloaded from a PC, or a website. Some of the stations are powered by solar energy, while others are connected to the electricity network. Five of them are mountainous, measure temperature and precipitation only. Five of the stations are placed in the Axios River basin (861Km^2), four in the basin of Vegoritida Lake (512Km^2) and one in the basin of Prespes Lake (326Km^2). The stations can be found in Figure 1. Data for the present study are taken from the AMS located at Amynteo (40^{o}41'40''N, 20^{o}40'40''E and 579.48m altitude); this belongs to the basin of Vegoritida Lake. Data at a 5 minute resolution were used from October 2002 until July 2009. Variables that were used for the estimation of the reference evapotranspiration with both methods are the mean (T_{mean}), minimum (T_{min}) and maximum (T_{max}) temperature, the minimum (RH_{min}) and maximum (RH_{max}) relative humidity, wind speed at 2m high (u_2), solar radiation (R_s), sunshine duration (n).

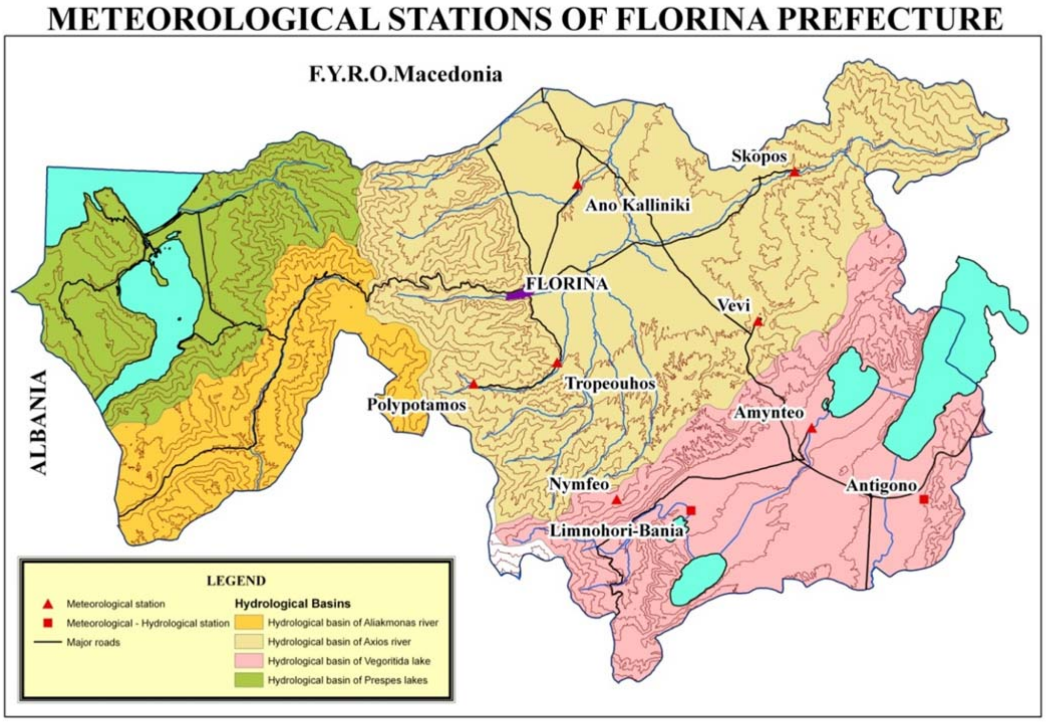

Figure 1. The Prefecture of Florina, Western Macedonia, Greece.
The study is presented in pink colour

4. RESULTS

The sensitivity coefficients are calculated, from equation 8, for three different periods:

- Annual scale, which is needed for water recourses management,
- Monthly scale, which is needed for the designing of irrigation systems and
- The irrigation period (May - September) for the designing of water reservoir.

The sensitivity analysis has been performed for all of the meteorological parameters that are needed in each method. The sensitivity analysis of evapotranspiration calculated from FAO 24 Penman was examined for mean temperature, minimum and maximum relative humidity, wind speed and solar radiation. Minimum and maximum temperature have not been used because they have strong correlation with the mean temperature and as Beven (1979) pointed out, the input data for sensitivity analysis should be uncorrelated; the correlation coefficient between mean-minimum and mean-maximum temperature is 0.953 and 0.975 respectively. The sensitivity analysis of evapotranspiration calculated from FAO Penman-Monteith was examined for mean temperature, minimum and maximum relative humidity, wind speed and solar radiation. FAO 24 Blaney-Criddle was examined for mean temperature, minimum relative humidity, wind speed and relative sunshine. FAO 24 Makkink was examined for mean temperature, mean relative humidity, wind speed and solar radiation. Hargreaves method was examined for mean temperature and solar radiation.

Table 1. Sensitivity coefficients

	Jan	Feb	Mar	Apr	May	Iun	Iul	Aug	Sept	Oct	Nov	Dec	Year	Irr.
	FAO 24 Penman													
T	0.36	0.35	0.17	0.11	0.09	0.08	0.05	0.06	0.09	0.12	0.18	0.27	0.29	0.1
RH_{min}	-0.14	-0.09	-0.05	-0.03	-0.02	-0.01	-0	-0.01	-0.03	-0.05	-0.12	-0.15	-0.05	-0.01
RH_{max}	-0.02	0	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	-0.01	-0.03	0.01	0.02
U_2	0.14	0.1	0.06	0.03	0.03	0.02	0.03	0.04	0.05	0.07	0.13	0.16	0.05	0.03
R_s	-0.02	0.16	0.24	0.29	0.22	0.18	0.13	0.12	0.21	0.2	0.07	-0.09	0.34	0.2
	FAO Penman – Monteith													
T	0.47	0.48	0.24	0.14	0.11	0.10	0.07	0.07	0.11	0.15	0.22	0.34	0.38	0.13
RH_{min}	-0.22	-0.16	-0.11	-0.07	-0.05	-0.03	-0.02	-0.03	-0.07	-0.11	-0.19	-0.23	-0.10	-0.04
RH_{max}	-0.07	-0.06	-0.04	-0.02	-0.01	-0.01	-0.01	-0.01	-0.02	-0.03	-0.05	-0.08	-0.03	-0.02
U_2	0.27	0.20	0.14	0.08	0.07	0.06	0.09	0.11	0.14	0.21	0.32	0.30	0.15	0.09
R_s	-0.01	0.06	0.13	0.18	0.15	0.12	0.09	0.08	0.13	0.11	0.01	-0.05	0.20	0.13
	FAO 24 BLANEY – CRIDDLE													
T	1.03	0.71	0.31	0.18	0.13	0.12	0.07	0.08	0.13	0.19	0.4	1.12	0.47	0.14
n/N	0.14	0.14	0.15	0.16	0.12	0.1	0.07	0.07	0.13	0.17	0.18	0.17	0.15	0.10
RH_{min}	-0.6	-0.3	-0.2	-0.2	-0.1	-0.1	-0.1	-0.1	-0.1	-0.19	-0.32	-0.66	-0.19	-0.10
U_d	0.32	0.22	0.12	0.07	0.06	0.04	0.05	0.05	0.07	0.09	0.15	0.36	0.09	0.05
	FAO 24 Makkink													
T	0.2	0.2	0.13	0.09	0.07	0.06	0.04	0.04	0.06	0.09	0.14	0.21	0.22	0.07
RH_{mean}	-0.13	-0.1	-0.09	-0.08	-0.07	-0.07	-0.05	-0.05	-0.08	-0.10	-0.11	-0.12	-0.09	-0.07
U_d	0.06	0.06	0.04	0.03	0.03	0.02	0.02	0.03	0.03	0.04	0.05	0.07	0.04	0.03
R_s	0.68	0.52	0.45	0.43	0.29	0.22	0.17	0.17	0.34	0.47	0.63	0.87	0.6	0.27
	Hargreaves													
T	-0.05	-0.24	0.84	0.25	0.14	0.12	0.08	0.09	0.13	0.2	0.56	-1.33	0.48	0.15
R_s	0.51	0.43	0.4	0.4	0.27	0.22	0.16	0.16	0.32	0.41	0.49	0.58	0.56	0.26

The sensitivity coefficients are calculated using the data of AMS Amynteo according to Equation 8. Table 1 presents the values of the sensitivity coefficients for the five methods and each temporal resolution (monthly, annual, irrigation period). From the monthly values we conclude that:

- The sensitivity coefficients present continuity during the year.
- Relative humidity presents negative sensitivity coefficients, as expected,
- The sensitivity coefficients present their highest values in winter for Temperature, Relative Humidity and wind speed and in summer for Solar Radiation.
- The sensitivity coefficients of wind speed present small alteration through the year.

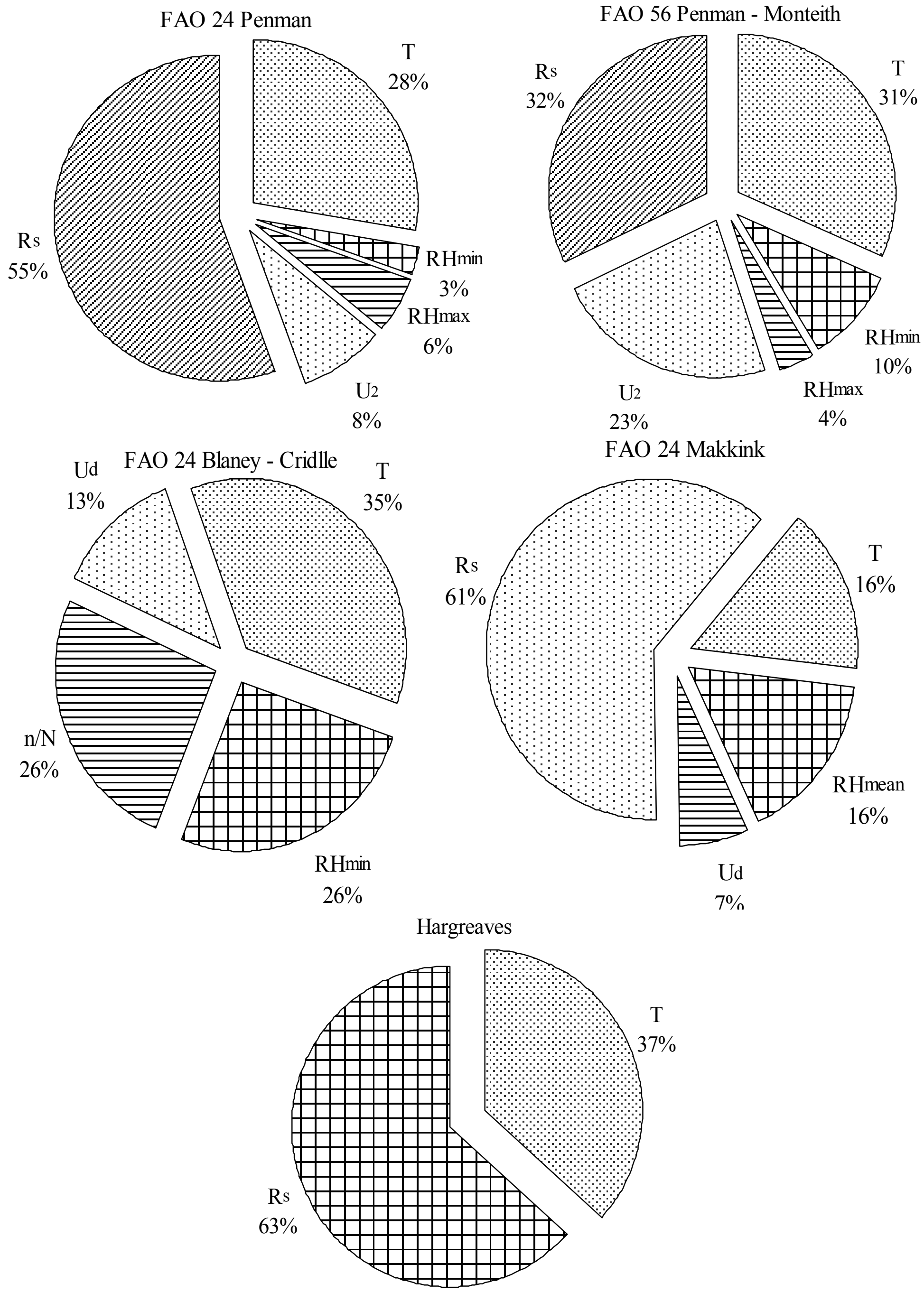

Figure 2. Relative influence of sensitivity coefficients on each evapotranspiration method for the irrigation period

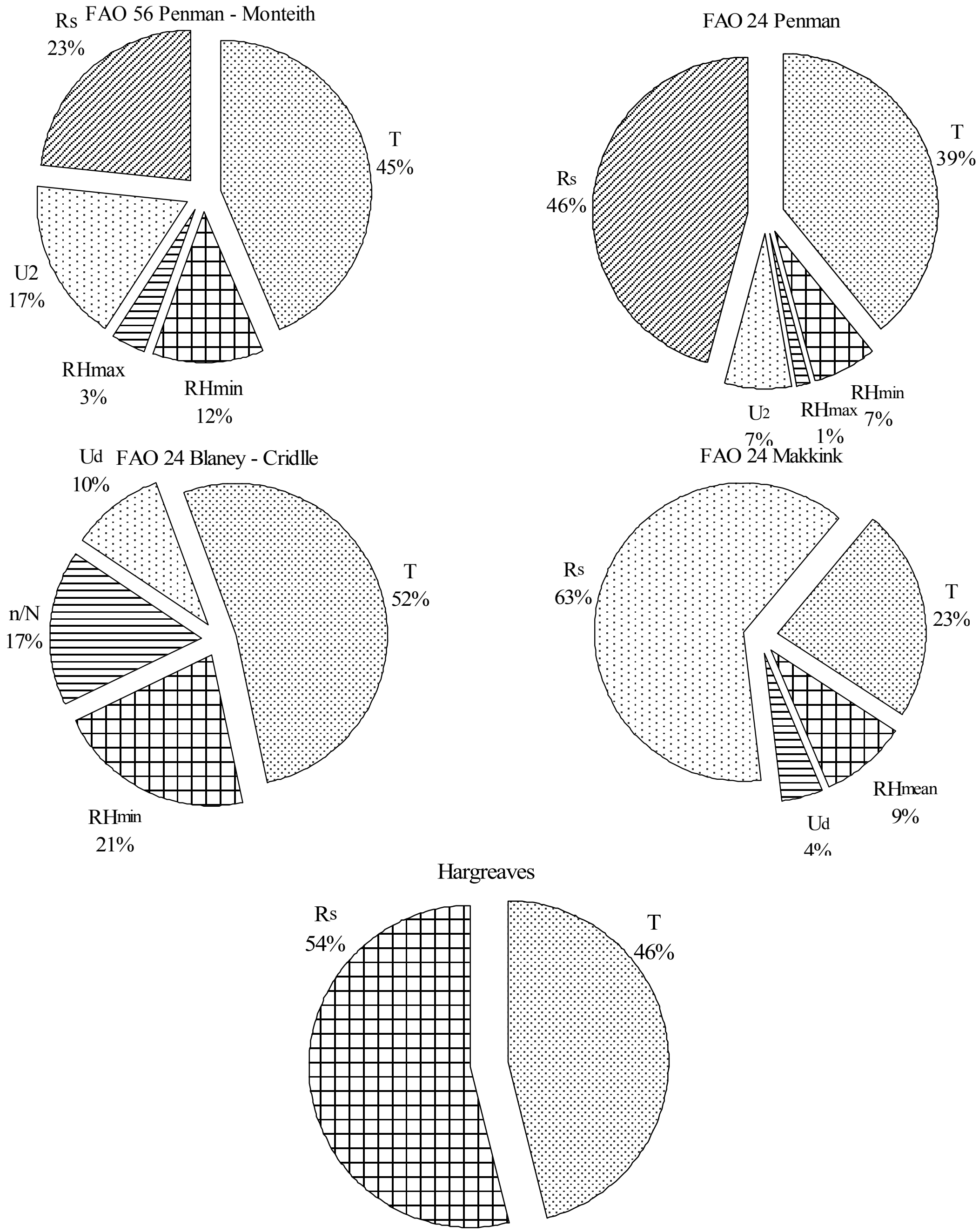

Figure 3. Relative influence of sensitivity coefficients on each evapotranspiration method at the annual scale

The relative influence of a sensitivity coefficient for each meteorological variable at the irrigation period are shown in Figure 2 whereas results at the annual scale are shown in Figure 3. The relative humidity increases in winter and decreases in summer. The sensitivity coefficient of temperature, for all the methods and the examined periods, ranges between 20% and 45% with an average of 30%. Wind speed is the meteorological parameter that influences the reference evapotranspiration about 10%. However when the FAO 56 Penman – Monteith method is used the impact is greater than 20%. The energy term, measured either by solar radiation or relative sunshine, seems to be the most important variable. The second most important parameter is the temperature. The average percentage of solar radiation, temperature, relative humidity and wind speed using the five methods and the periods of interest is 41, 30, 15, and 13% respectively. Overall, the comparison between the five different methods shows that the solar radiation and the temperature are the two most significant parameters to estimate evapotranspiration, whereas the relative humidity and the wind speed did not have such a significant effect.

5. CONCLUSIONS

Sensitivity analysis, in this paper, is conducted to assess the influence of the key meteorological variables on reference evapotranspiration as this is estimated via five different methods (the FAO56 Penman-Monteith, the FAO 24 Penman, the FAO24 Blaney-Cridlle, the FAO24 Makkink and the Hargreaves). To address that a new sensitivity coefficient is applied which uses the standard deviation of the variables. The meteorological variables examined are temperature, relative humidity, wind speed, solar radiation, sunshine duration. The sensitivity coefficients are calculated at monthly, annual scale and during the irrigation period. The comparison is performed for the month of water peak demand (July), the irrigation period and the year for each evapotranspiration method. Results using data from the AMS of Amynteo, Florina – Greece, show that solar radiation and the energy term (n/N) seem to be the most important variables, followed by temperature, while, wind speed and relative humidity are not important climatic parameters.

REFERENCES

Allen R.G. and Pruitt W.O., (1991). FAO-24 reference evapotranspiration factors, *Journal Irrigation and Drainage Engineering*, ASCE, **117**(IR5), 758-773.

Allen R.G., Pereira L.S., D. Raes and M. Smith, (1998). Crop Evapotranspiration Guidelines for Computing Crop Water Requirements. FAO Irrigation and Drainage Paper, No 56.

A.S.C.E. Standardization of Reference Evapotranspiration Task Committee, (2005). The ASCE standardized reference evapotranspiration equation. Environmental and Water Resources Institute of the American Society of Civil Engineers.

Ampas V., (2010). Research and estimation of meteorological parameters with direct impact on agriculture. Ph.D. Thesis. Aristotle University of Thessaloniki, Greece.

Ampas V., Baltas E. and Papamichail D., (2007). Comparison of different methods for the estimation of the reference crop evapotranspiration in the Florina region, *WSEAS Journal*, **2**(12),1449-1454.

Ampas V., Baltas E., Papamichail D. and Georgiou P., (2009). Sensitivity analysis of FAO Penman – Monteith method to key climatic variables in the Florina region. Proc of Common conf. 11th HHA – 7th GCWRM. Volos 2009, pp355-362.

Babajimopoulos Ch., Antonopoulos B., Grigoriadis D. and Ilias A., (1992). *Sensitivity analysis of the Penman method.* proceedings of 5th conference of H.Y.U., p. 132-140.

Beven K., (1979). A sensitivity analysis of the penman Monteith actual evapotranspiration estimates, *Journal of Hydrology*, **44**, 169-190.

Coleman G. and DeCoursey G.D., (1976). Sensitivity analysis applied to some evaporation and evapotranspiration models, *Water Resour. Res.*, **12**(5), 873-879.

Doorenbos J. and Pruitt W.O., (1977). Guidelines for predicting crop water requirements. FAO Irrigation and Drainage Paper No 24, 2nd ed., FAO Rome, Italy.

Gong L., Xu C., Chen D., Haldin S. and Chen Y.D., (2006). Sensitivity of the Penman-Monteith evapotranspiration to key climatic variables in the Changjiang (Yangtze River) basin, *Journal of Hydrology*, **329**, 620-629.

Irmak S., Payero J.O., Martin D.L., Irmak A. and Howell A.T., (2006). Sensitivity coefficients of standardized daily ASCE Penman –Monteith equation, *J. Irrig. Drain. E.-ASCE*, **132**(6), 564-578.

McCuen H.R., (1974). A sensitivity and error analysis of procedures used for estimating evapotranspiration. *Water Resour. Bull.*, **10**(3), 486-498

Saxton K.E., (1975). Sensitivity analysis of the combination evapotranspiration equation, *Agricultural Meteorology*, **15**, 343-353.