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DECISION MAKING IN WATER RESOURCES WITH FUZZY INFORMATION

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ABSTRACT

The multiple objectives optimization in water resources planning consists in trading multiple and conflicting objectives, forming a complex and dynamic process. In the last four decades multi-objective decisions based on fuzzy sets have been evolved and considerable research spawned into the application of fuzzy subsets. Multiobjective decisions problems with uncertainty require: a) evaluating how well each alternative or choice satisfies each objective and b) combining the objectives into an overall objective or decision function D for the selection of the best alternative. In particular when one has a) a universe of n alternatives X={X_1, X_2,...X_n} and a set of p objectives (criteria) A={A_1,A_2,….A_p} to be satisfied, the overall objective is D=A_1 and A_2 …….and A_p, given by the intersection of all the objectives, D= $A_1 \cap A_2 ...... \cap A_p$ and one is seeking solutions satisfying D, with $\mu_D(X^*)=\max(\mu_D(X))$, where $\mu_D(X)$ is the grade of membership that the decision function D has for each alternative. An application of the above theory concerns the decision of selecting the most appropriate from five dams and their corresponding reservoirs in Néstos watershed (Alternatives AB, AD, AR, BA, and MA). The criteria set is A= {A_1=cost of the dam, A_2=environmental impact, A_3=Hydroelectric power production, A_4=flood protection) and finally the importance set is: P= {b_1, b_2, b_3, b_4}.

KEYWORDS: multiple objectives optimization, fuzzy analysis, membership function, Néstos reservoirs, preferences(importance)

1. INTRODUCTION

Most problems of today's concern are characterized by multiple objectives, criteria or goals according to which the best solution is to be found. Because of the conflicting character and noncommensurability of such criteria, a concept of multiple objective *satisficing rather than optimal solution* is more useful for their analysis (Zeleny, 1982). A water resources problem belongs to this category and especially today the decrease of available water resources and the degradation of water quality as well as the rapid increase of population combined with the growth of human activities, have forced engineers to contemplate and propose even more comprehensive, complex, and ambitious plans for water resources systems. The application of systems methods such as mathematical optimization and simulation can satisfactory aid to the definition, evaluation and selection of water resources investments, design and policies.

Water resources planning must take into account multiple users, multiple purposes, and multiple objectives. Water engineers and planners should develop a number of reasonable alternatives for public officials to consider. They should also evaluate the economic, environmental, political and social impacts that might result from each alternative. So it is impossible to develop a single objective that satisfies all interests, all adversaries, and all political and social viewpoints (Loucks *et al*., 1981; Iliadis and Maris , 2007).

Multiobjective or multicriterion optimization in water resources planning consists in trading multiple and conflicting objectives, forming a complex and dynamic process (Zeleny, 1982). A water resources problem may have various design levels, from a simple structure to the construction of a complex system with managerial and engineering elements. For the design of a complex structure (for example a surface reservoir) a variety of natural phenomena are incorporated in the design and increase the conflicting objectives. Many methods have been evolved for the evaluation of the above problems: ELECTRE I (Roy, 1968), AHP (Saaty, 1975; 1977; 1980), ELECTRE II and ELECTRE III (Belton and Stewart, 2001), Compromise Programming (Zeleny, 1982), MCQA-I and MCQA-II (Duckstein *et al*., 1991).

As decision sciences become more and more involved in both humanistic and complex systems, fuzziness becomes a prevalent phenomenon in describing these systems. Zadeh (1973) calls these fuzziness as the "Principal of Incompatibility" : *as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior, diminishes until a threshold is reached beyond which precision and significance become almost mutually exclusive characteristics and finally the closer one looks at a real-world problem, the fuzzier becomes its solution*. In the last four decades multi-objective decisions based on fuzzy sets have been evolved and considerable research spawned into the application of fuzzy subsets. (Belmann and Zadeh, 1970; Yager, 1975; 1977; 1978; 1981), Kecman, 2001; Ross, 2004; Cox, 2007; Iliadis, 2007). Multiobjective decisions problems with fuzziness require the choice of one element from a set {X} of possible alternatives, given a collection of criteria of concern to the decision maker. So two problems arise: a) evaluating how well each alternative or choice satisfies each objective and b) combining the objectives into an overall objective or decision function D for the selection of the best alternative. In particular when one has a) a universe of n alternatives $X=\{X_1, X_2,\ldots, X_n\}$ and a set of p objectives (criteria) $A=\{A_1,A_2,\ldots.A_p\}$ to be satisfied, the overall objective is $D=A_1$ and A_2 …….and A_p, leading to the intersection of all the objectives, $D= A_1 \cap A_2 \ldots\ldots \cap A_p$ and one is seeking solutions satisfying D. The *maximizing decision* X* will then be the alternative satisfying $\mu_D(X^*)=\max(\mu_D(X))$, where $\mu_D(X)$ is the grade of membership that the decision function D has for each alternative.

In this paper an application of the above theory concerning the decision of selecting one out of five dams (five alternatives) is presented, satisfying four objectives: a) The cost of the dam (A_1), b) the environmental impact of each dam (A_2), c) the produced hydroelectric power (A_3), and d) the protection of floods (A_4). Additionally one also has to rank the preferences for these objectives on the unit interval (b_1,b_2,b_3,b_4). The five dams (alternatives AB, AD, AR, BA, and MA) belong to the Néstos watershed in Greece.

2. MATHEMATICAL MODEL

2.1 Preliminaries

Fuzziness, as handled in fuzzy logic, can refer to various types of vagueness and uncertainty but particularly to the vagueness related to human linguistics and thinking, differing from the uncertainty of the Probabilistic Theory. In Boolean logic the boundaries of a set are clearly defined and it is evident whether an object belongs to a set or not. It is described by a binary function, the characteristic function, taking the value 0 when an element x does not belong to the set A and the value 1 when it does. On the contrary, in fuzzy logic, it is possible for elements to belong partially to a set (Zadeh, 1965).

Definition 1. Fuzzy set

If X is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ on X is a set of ordered pairs: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid \quad x \in X\}$, $\mu_{\tilde{A}} \in [0,1]$, where $\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in $\tilde{A}$, that maps X to the membership space M. When M contains only two points 0 and 1, A is nonfuzzy and its membership function becomes identical with the characteristic function of a nonfuzzy set. The symbol ~ will be referred to as a fuziffier (Zadeh, 1965).

Definition 2. α-level cut

The (crisp) set of elements belonging to the fuzzy set $\tilde{A}$ at least to the degree α, is called the α-level set, or the α-cut : $A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}$. If $A'_\alpha = \{x \mid \mu_A(x) > \alpha\}$, it is called "strong α- cut".

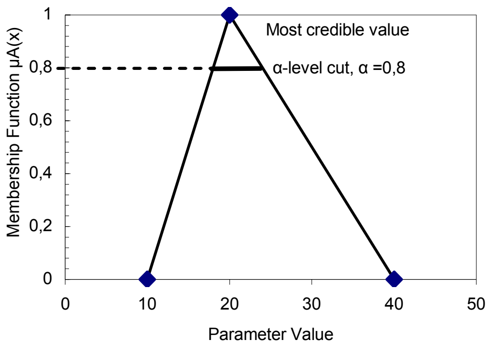

Figure 1. A fuzzy number and an α- cut

Definition 3. Convex fuzzy set

A fuzzy set is convex if:

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2), \quad x_1, x_2 \in X, \lambda \in [0,1]$$

Also, a fuzzy set can be convex if all *α*-level sets are convex.

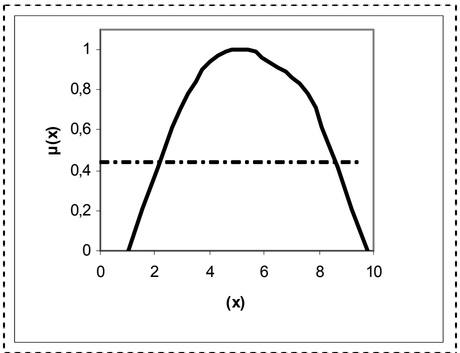

*Figure 2.*Convex fuzzy set

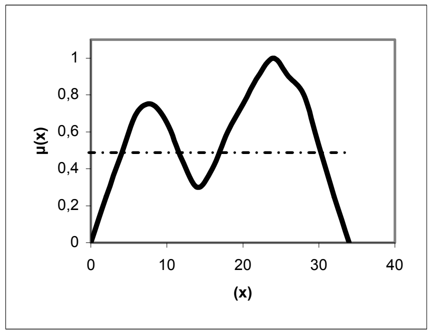

Figure 3. Nonconvex fuzzy set

Definition 4. Fuzzy numbers

A fuzzy number $\tilde{M}$ is a convex normalized set $\tilde{M}$ of the real line $\Re$ such that:

1. It exists exactly one $x_0 \in \Re$ with $\mu_{\tilde{M}}(x_0) = 1$ (*x_0* is called the mean value of $\tilde{M}$)
2. $\mu_{\tilde{M}}(x) = 1$ is piecewise continuous.

Nowadays, this definition is very often modified. For the sake of computational efficiency, trapezoidal membership functions are often used. A triangular fuzzy number is, of course, a special case of this.

Multiobjective decisions problems with uncertainty

In the last four decades multi-objective decisions based on fuzzy sets have been evolved and considerable research spawned into the application of fuzzy subsets. Multiobjective decisions problems with uncertainty require the choice of one element from a set {X} of possible alternatives, given a collection {A} of criteria of concern to the decision maker. So two problems arise:

- Evaluating how well each alternative or choice satisfies each objective.
- Combining the objectives into an overall objective or decision function D, satisfying all the objectives, and from which we select the best alternative.

In particular when one has a collection {A}={A_1, A_2, A_3 ...., A_p} of objectives to be satisfied, the overall objective is D=A_1 and A_2 …….and A_p, and one is seeking solutions satisfying A_1 and A_2 and A_3….etc. As the mathematical form for the “and” operation in fuzzy logic is the minimum operator (the intersection of fuzzy subsets) the decision function is:

$$D = A_1 \cap A_2 \cap A_3 ...... \cap A_p \quad (1)$$

and the corresponding membership function μ_D:

$$\mu_D = \mu_{A_1} \wedge \mu_{A_2} \wedge \mu_{A_3} \wedge ...... \wedge \mu_{A_p} . \quad (2)$$

Comment. It should be noted that Bellman and Zadeh (1970) identified the connective "and" with the conjunction symbol $\wedge$, interpreting "and" in a "hard" sense, that is, they do not allowed any tradeoffs between the memberships μ_{A_i}, $i = 1,.....p$ (non-interactive "and"). In some cases a softer interpretation of "and" corresponds to forming the algebraic product of memberships μ_{A_i}, $i = 1,.....p$ (Interactive "and"). From the mathematical as well as practical point of view, the identification of "and" with $\wedge$, is preferable to its identification with the product. The above operators (min and product) belong to a general class of operators for the intersection and union of fuzzy sets, called triangular norms (t-norms) and conorms (t-conorms or s-norms), (Zimmermann, 1996). Yager (1981) refers that the Bellman-Zadeh optimal solution leads always to a Pareto optimal solution and Ross (2004) refers it as optimum decision. In what follows "and" will be understood to be a hard "and", and the above expression becomes:

$$\mu_D = \mathrm{Min}(\mu_{A_1}, \mu_{A_2}, \mu_{A_3}, ...\mu_{A_p}) . \quad (3)$$

The ***optimal solution*** (Zimmermann, 1992; 1996; Ross, 2004) is Max $\{\mu_D\}$, that is:

$$\mathrm{Max}\{\mu_D\} = \mathrm{Max}\{\min(\mu_{A_1}, \mu_{A_2}, \mu_{A_3}, ...\mu_{A_p})\} . \quad (4)$$

The Bellman-Zadeh approach to multiobjective decision making has the advantage of requiring only an ordinal evaluation of the preference information, but it has also the disadvantage of not allowing one to include the fact that the objective differ in importance. For the case that we define a set of different importance {P}={b_1, b_2, b_3,…..b_p}, the overall decision function D takes a more general form:

$$D = M(A_1, b_1) \cap M(A_2, b_2) \cap M(A_3, b_3)...... \cap M(A_p, b_p) , \quad (5)$$

where $M(A_i,b_i)$ is a new function involving objective A_i and its importance b_i. Yager(1977,1978) extended the Bellman-Zadeh approach in order to include the importance of the various objectives(preferences), using Saaty theory, (Saaty, 1975;1977), who has developed a procedure for obtaining a ratio scale for group of elements, based upon a paired comparison of each of the elements. Yager (1977) applied the above method of Saaty, assigning to each objective a power indicative of its importance and then raising each fuzzy set to its appropriate power. These powers were obtained by getting the eigenvector of the maximum eigenvalue of a matrix of paired comparisons of the objectives and the overall decision function D takes the form:

$$D = A_1^{\alpha_1} \cap A_2^{\alpha_2} \cap A_3^{\alpha_3} ...... \cap A_p^{\alpha_p} , \quad (6)$$

and the corresponding membership function μ_D:

$$\mu_D = \mathrm{Min}(\mu_{A_1}{}^{\alpha_1}, \mu_{A_2}{}^{\alpha_2}, \mu_{A_3}{}^{\alpha_3}, ...\mu_{A_p}{}^{\alpha_p}) . \quad (7)$$

As he notes (Yager, 1981) in some cases it may be very difficult for a decision maker to supply information about α_i and he extended his methodology, applying the logical implication:

$$b_i \rightarrow A_i = \neg b_i \vee A_i = b_i' \vee A_i, \quad \neg b_i = b_i' = 1 - b_i$$

and showing that $A_i^{b_i}$ and $b_i' \vee A_i$ are both acceptable operations for implication, that is, they both generally act in the same manner. Now the decision function D becomes:

$$D = \bigcap_{i=1}^{p} (b_i' \cup A_i) . \quad (8)$$

The maximum of the new function D becomes:

$$\mathrm{Max}\{\mu_D\} = \mathrm{Max}\{\min\{\mu_{C_1}, \mu_{C_2}, \mu_{C_3}, ...\mu_{C_p}\}\} , \quad (9)$$

where $C_i = (1 - b_i) \cup A_i$.

3. APPLICATIONS

3.1 Application area

The Néstos River flows in south western Bulgaria and western Thrace, Greece (Mylopoulos *et al.*, 2004). The Néstos rises on Kolarov peak of the Rila Mountains of the north western Rhodope (Rodopi) Mountains. The river's upper confluents separate the Rila and Pirin ranges from the main Rhodope massif. Crossing the Bulgarian frontier into Greece, the Néstos divides Greek Macedonia from Greek Thrace. About 110 km of the river flow through Bulgaria and about 130 km through Greece. The total catchment's area of the river is about 5 800 km^2, of which ~2 800 km^2 (48%) belong to Greece. From just west of Stavroúpolis to its mouth on the Aegean Sea, 150 miles (240 km) from its source, it forms the boundary between Kaválla and Xánthi departments. Above Paranéstion, however, the river is confined to inaccessible gorges, as it traverses the sparsely populated, mountainous Dráma department. West of Xánthi , Néstos reaches the marshy, alluvial coastal plain of Chrysopolis.

The area mainly consists of metamorphic rocks and marbles (it is believed that in the past they constituted limestone reefs), while the river bed is composed of sedimentary rocks and alluvial deposits (recent fluvial deposits). The Nestos River threads through a large gorge displaying steep rocky slopes and riverside vegetation encompassing well formed clumps of trees and stands of Salix alba, S. fragilis, S. amplexicaulis, S. eleagnos, Populus alba, Pinus nigra. The local habitat also exhibits a sparse growth (individuals) of Platanus orientalis and Alnus glutinosa. From a geological point of view, the area belongs to the Rodopi mass. The climatic type ranges between the Mediterranean and continental type of climate.

Mpaka (2006), applying compromise programming, studied and suggested the construction of five dams (five alternatives, X_1=AB, X_2=AD, X_3=AR, X_4=BA, and X_5=MA)) with their corresponding reservoirs in Néstos watershed (Fig. 1): 1) The dam of Agia Barbara (AB), 2) The dam of Ano Potamaki (AD), 3) The dam of Arkoudorema (AR), 4) The dam of Bathyrema (BA) and 5)The dam of Mavrorema. (MA)

Figure 1. Néstos Catchment area.

3.2. Results

The research concerning the five sub catchments focused in an estimation of the cost of each dam (€), the volume (m^3) and the area (m^2) of the reservoir, and the potential hydroelectric power (in kW) (Mpaka, 2006). The decision maker has to define four objectives (A_1, A_2, A_3, A_4) (Table 1), that impact the decision: a) The small cost of the dam (A_1), b) the environmental impact of each dam (A_2), c) the produced hydroelectric power (A_3), and d) the protection of floods (A_4). Besides one also decides to rank the preferences for these objectives on the unit interval (b_1,b_2,b_3,b_4).

Hence one sets up the problem as follows: a) X = {AB, AD, AR, BA, MA }={ X_1, X_2, X_3, X_4, X_5}, b) A = {cost, env, power, flood} = (A_1, A_2, A_3, A_4), c) P = { b_1,b_2,b_3,b_4}.

Table 1. Objectives for each dam

a/a	AB	AP	AR	BA	MA
Cost	0.286	0.169	0.197	0.122	0.225
Env	0.171	0.180	0.321	0.265	0.063
Power	0.126	0.116	0.328	0.358	0.071
Flood	0.131	0.160	0.319	0.328	0.063

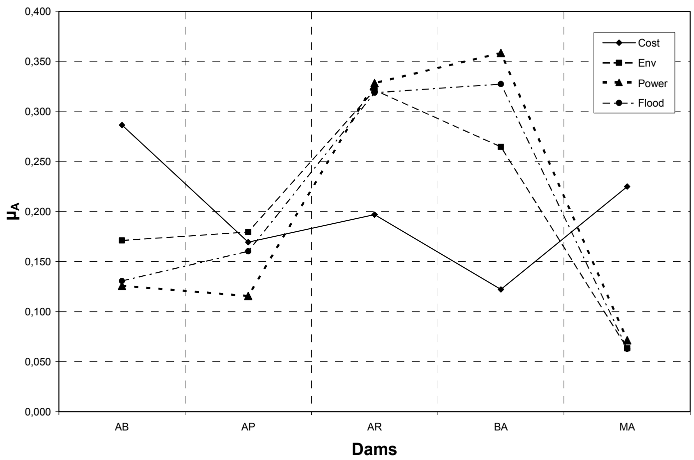

Figure 2. Memberships for each alternative with respect to the objectives

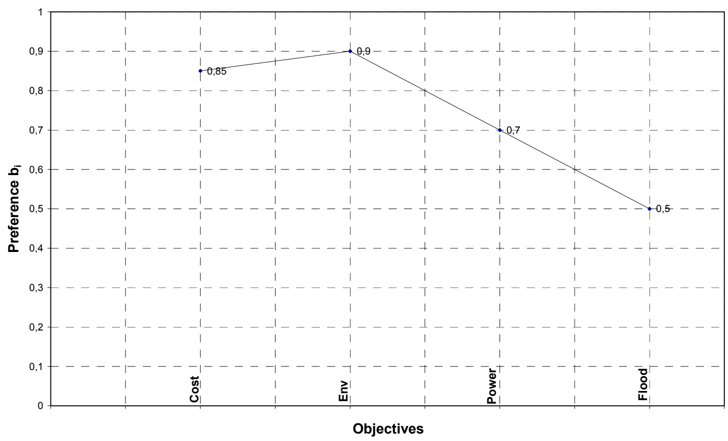

Figure 3. Importance of each objective

The ratings of the dams with respect to the objectives are given here expressed in Zadeh's notation (Zadeh, 1965, 1973):

$$\tilde{A}_1 = \{\frac{0.286}{AB} + \frac{0.169}{AR} + \frac{0.197}{BA} + \frac{0.122}{BA} + \frac{0.225}{MA}\}$$
$$\tilde{A}_2 = \{\frac{0.171}{AB} + \frac{0.180}{AR} + \frac{0.321}{BA} + \frac{0.265}{BA} + \frac{0.063}{MA}\}$$
$$\tilde{A}_3 = \{\frac{0.126}{AB} + \frac{0.116}{AR} + \frac{0.328}{BA} + \frac{0.358}{BA} + \frac{0.071}{MA}\}$$
$$\tilde{A}_4 = \{\frac{0.131}{AB} + \frac{0.160}{AR} + \frac{0.319}{BA} + \frac{0.328}{BA} + \frac{0.063}{MA}\}$$

Now, one wishes to determine the sensitivity of the optimum solution to the preference ratings.(The preference ratings were defined subjectively, Ross, 2004). From these preferences (Fig. 3) the following calculations result:

$b_1 = 0.85$, $b_2 = 0.9$, $b_3 = 0.7$, $b_4 = 0.5$

$b_1' = 0.15$, $b_2' = 0.1$, $b_3' = 0.3$, $b_4' = 0.5$

$$\mu_{\tilde{D}(X_1)} = (b_1' \cup \tilde{A}_1) \cap (b_2' \cup \tilde{A}_2) \cap (b_3' \cup \tilde{A}_3) \cap (b_4' \cup \tilde{A}_4)$$
$$= (0.15 \vee 0.286) \wedge (0.10 \vee 0.171) \wedge (0.3 \vee 0.126) \wedge (0.5 \vee 0.131) =$$
$$= 0.286 \wedge 0.171 \wedge 0.3 \wedge 0.5 = 0.171$$

$$\mu_{\tilde{D}(X_2)} = (b_1' \cup \tilde{A}_1) \cap (b_2' \cup \tilde{A}_2) \cap (b_3' \cup \tilde{A}_3) \cap (b_4' \cup \tilde{A}_4)$$
$$= (0.15 \vee 0.169) \wedge (0.10 \vee 0.180) \wedge (0.3 \vee 0.116) \wedge (0.5 \vee 0.160) =$$
$$= 0.169 \wedge 0.180 \wedge 0.3 \wedge 0.5 = 0.169$$

$$\mu_{\tilde{D}(X_3)} = (b_1' \cup \tilde{A}_1) \cap (b_2' \cup \tilde{A}_2) \cap (b_3' \cup \tilde{A}_3) \cap (b_4' \cup \tilde{A}_4)$$
$$= (0.15 \vee 0.197) \wedge (0.10 \vee 0.321) \wedge (0.3 \vee 0.328) \wedge (0.5 \vee 0.319) =$$
$$= 0.197 \wedge 0.321 \wedge 0.328 \wedge 0.5 = 0.197$$

$$\mu_{\tilde{D}(X_4)} = (b_1' \cup \tilde{A}_1) \cap (b_2' \cup \tilde{A}_2) \cap (b_3' \cup \tilde{A}_3) \cap (b_4' \cup \tilde{A}_4)$$
$$= (0.15 \vee 0.122) \wedge (0.10 \vee 0.265) \wedge (0.3 \vee 0.358) \wedge (0.5 \vee 0.328) =$$
$$= 0.150 \wedge 0.265 \wedge 0.358 \wedge 0.5 = 0.150$$

$$\mu_{\tilde{D}(X_5)} = (b_1' \cup \tilde{A}_1) \cap (b_2' \cup \tilde{A}_2) \cap (b_3' \cup \tilde{A}_3) \cap (b_4' \cup \tilde{A}_4)$$
$$= (0.15 \vee 0.225) \wedge (0.10 \vee 0.063) \wedge (0.3 \vee 0.071) \wedge (0.5 \vee 0.063) =$$
$$= 0.225 \wedge 0.100 \wedge 0.3 \wedge 0.5 = 0.100$$

$$\text{Max}\{\mu_{\tilde{D}}\} = \text{Max}\{\mu_{\tilde{D}(X_1)}, \mu_{\tilde{D}(X_2)}, \mu_{\tilde{D}(X_3)}, \mu_{\tilde{D}(X_4)}, \mu_{\tilde{D}(X_5)}\} =$$
$$= \text{Max}\{0.171,\ 0.169,\ 0.197,\ 150,\ 100\} = 0.197$$

Consequently one chooses the third alternative, which means the dam of Arkoudorema.

4. CONCLUSIONS

A problem of multiobjective decision making in water resources management has been presented here, in the catchment area, of Néstos having five alternatives (five dams) to consider for selecting the optimum solution. Applying the Yager (1981) methodology, one concluded that the best solution is the dam of Arkoudorema with the following criteria:The cost of the dam 0,197, (b_1=0.85), b) the environmental impact of the dam 0.321, (b_2=0.90), c) the produced hydroelectric power 0.328, (b_3=0.70), and d) the protection of floods 0.319, (b_4=0.50). This method is very easy for engineering applications and one can proceed, making simple numerical calculations. But as is pointed out Ross (2004), the objectives and the preferences are not known with precision and are rather subjective. Much of this imprecision is not measurable or random and it can be due to vague, ambiguous, or fuzzy information. According to Carlson and Fuller (1996), decision making in practice has shown that fuzzy logic allows decision making with estimated values in spite of incomplete information and even if a decision may not be correct, it can be improved later when additional information is available. It is also possible in the above problem to have a different set of importance and the decision maker will then choose an other best solution.

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